Lehrstuhl A für Mathematik
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Calculus and Linear Algebra

Test

Exercise 1 (3 points).

Let $z = \frac{3}{5} - \frac{4}{5}i \in \mathbb{C}$.

- Compute $|z|$.
- Which of the following numbers is z^{-1}?

a) $w = 1 + i$ b) $w = 1$ c) $w = \frac{3}{5} + \frac{4}{5}i$ d) $w = \dfrac{5 + 14i}{13}$

Exercise 2 (4 points)(We subtract a point per false answer)

Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 4 & 7 & 2 \\ 2 & -2 & 2 \end{pmatrix}, \; B = \begin{pmatrix} 3 & -1 \\ 4 & 7 \\ 3 & 2 \end{pmatrix}, \; C = \begin{pmatrix} 4 & 8 & 7 \\ 1 & -2 & 1 \end{pmatrix}$$

For the following matrix products, determine whether they are well-defined:
a) AC b) BCA c) $B^T A$ d) CC^T

Exercise 3 (2 points).

Let $z \in \mathbb{C} \setminus \{0\}$ have polar coordinates (r, α). What are the polar coordinates of $z^{-1}\bar{z}$?

a) $(r, 2\alpha)$ b) (r^2, α) c) $(1, -2\alpha)$ d) $(2\alpha, 1)$.

Exercise 4 (4 points)(We subtract 2 points per false answer).

For each of the following systems of linear equations, decide whether it has a unique solution.

a)
$$\begin{array}{rrrcl} 2x & +3y & -z & = & 5 \\ -4x & +8y & +2z & = & 2 \\ 4x & +6y & -2z & = & 10 \end{array}$$

b)
$$\begin{array}{rrrcl} 2x & +3y & -z & = & 5 \\ & y & +2z & = & 1 \\ & & z & = & 2 \end{array}$$

Exercise 5 (3 points).

Compute the determinant of the matrix

$$\begin{pmatrix} a & a^2 & 1+a^3 \\ 0 & a-1 & a^2 \\ 0 & 0 & a-2 \end{pmatrix} .$$

For which values of a is the matrix invertible?

Exercise 6 (4 points).

Consider the line

$$\mathbb{L} = \{\mathbf{c} \in \mathbb{R}^2 : \mathbf{c} \cdot (1,3)^T = 10\} \ ?$$

(a) Which of the following points belong to $\mathbb{L}$?
i) $(3,-1)^T$ ii) $(1,3)^T$ iii) $(7,-1)^T$ iv) $(-2,4)^T$

(b) Compute the distance of the point $(2,5)^T$ to $\mathbb{L}$.