



Calculus and Linear Algebra

Test

Exercise 1 (3 points)

Consider the functions $f(x) = e^{2x}$, $g(x) = \sin(x + 2)$ and $h(x) = \frac{1}{x}$. Which of the following expressions is $(g \circ f \circ h)(x)$?

- a) $\sin(e^{2x}) + 2$ b) $e^{2/\sin(x+2)}$ c) $\sin(e^{2/x} + 2)$ d) $\sin(e^{2/x+2})$

Exercise 2 (4 points) (We subtract a point per false answer)

For each of the following sequences $(a_n)_{n \in \mathbb{N}}$, decide whether it converges in \mathbb{R} .

- | | convergent | divergent |
|--------------------------------------|--------------------------|--------------------------|
| a) $a_n = 2^n + 2^{-n}$ | <input type="checkbox"/> | <input type="checkbox"/> |
| b) $a_n = 2^{-n} - \cos(1/n)$ | <input type="checkbox"/> | <input type="checkbox"/> |
| c) $a_n = \frac{5n^7 - 1}{3n^5 + 1}$ | <input type="checkbox"/> | <input type="checkbox"/> |
| d) $a_n = \frac{5n^5 - 1}{3n^7 + 1}$ | <input type="checkbox"/> | <input type="checkbox"/> |

Exercise 3 (4 points) (We subtract a point per false answer)

For each of the following series, decide whether it converges in \mathbb{R} .

- | | convergent | divergent |
|---|--------------------------|--------------------------|
| a) $\sum_{k=0}^{\infty} \frac{1}{k!}$ | <input type="checkbox"/> | <input type="checkbox"/> |
| b) $\sum_{k=1}^{\infty} \frac{1}{k^3} + \frac{1}{k!}$ | <input type="checkbox"/> | <input type="checkbox"/> |
| c) $\sum_{k=1}^{\infty} k + \frac{1}{k^3}$ | <input type="checkbox"/> | <input type="checkbox"/> |
| d) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$ | <input type="checkbox"/> | <input type="checkbox"/> |

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Exercise 4 (3 points) (We subtract a point per false answer)

Decide, whether the following statements are true for all continuously differentiable functions $f, g : [a, b] \rightarrow \mathbb{R}$.

- | | True | False |
|--|--------------------------|--------------------------|
| a) If $f'(x) = 0$, then $(f \cdot g)'(x) = 0$. | <input type="checkbox"/> | <input type="checkbox"/> |
| b) If $f'(a)f'(b) < 0$, then there exists a local extremum $x \in (a, b)$. | <input type="checkbox"/> | <input type="checkbox"/> |
| c) If $f'(a)f'(b) > 0$, there exists no local extremum $x \in (a, b)$. | <input type="checkbox"/> | <input type="checkbox"/> |

Exercise 5 (6 points)

Compute the derivatives of the following functions:

- a) $f(x) = \sqrt{\sin(x^3)}$ b) $g(x) = \ln(\cos(x^3 + 2))$ c) $h(x) = \exp(2 \ln(x)) + \exp(\cos(x) + x^2 + 1)$