Lehrstuhl A für Mathematik
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Aachen, den January 8, 2009

Calculus and Linear Algebra

Test

Exercise 1 (3 points)

Consider the functions $f(x) = e^{2x}$, $g(x) = \sin(x+2)$ and $h(x) = \frac{1}{x}$. Which of the following expressions is $(g \circ f \circ h)(x)$?

a) $\sin(e^{2x}) + 2$ b) $e^{2/sin(x+2)}$ c) $\sin(e^{2/x} + 2)$ d) $\sin(e^{2/x+2})$ **Answer:** c)

Exercise 2 (4 points) (We subtract a point per false answer)

For each of the following sequences $(a_n)_{n\in\mathbb{N}}$, decide whether it converges in $\mathbb{R}$.

	convergent	divergent
a) $a_n = 2^n + 2^{-n}$	$\square$	$\surd$
b) $a_n = 2^{-n} - \cos(1/n)$	$\surd$	$\square$
c) $a_n = \frac{5n^7 - 1}{3n^5 + 1}$	$\square$	$\surd$
d) $a_n = \frac{5n^5 - 1}{3n^7 + 1}$	$\surd$	$\square$

Exercise 3 (4 points) (We subtract a point per false answer)

For each of the following series, decide whether it converges in $\mathbb{R}$.

	convergent	divergent
a) $\sum_{k=0}^{\infty} \frac{1}{k!}$	$\surd$	$\square$
b) $\sum_{k=1}^{\infty} \frac{1}{k^3} + \frac{1}{k!}$	$\surd$	$\square$
c) $\sum_{k=1}^{\infty} k + \frac{1}{k^3}$	$\square$	$\surd$
d) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$	$\surd$	$\square$

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Exercise 4 (3 points) (We subtract a point per false answer)

Decide, whether the following statements are true for all continuously differentiable functions $f, g : [a, b] \to \mathbb{R}$.

	True	False
a) If $f'(x) = 0$, then $(f \cdot g)'(x) = 0$.	☐	√
b) If $f'(a)f'(b) < 0$, then there exists a local extremum $x \in (a, b)$.	√	☐
c) If $f'(a)f'(b) > 0$, there exists no local extremum $x \in (a, b)$.	☐	√

Exercise 5 (6 points)

Compute the derivatives of the following functions:

a) $f(x) = \sqrt{\sin(x^3)}$ b) $g(x) = \ln(\cos(x^3 + 2))$ c) $h(x) = \exp(2\ln(x)) + \exp(\cos(x) + x^2 + 1)$

$f'(x) = \frac{3x^2 \cos(x^3)}{2\sqrt{\sin(x^3)}}$

$g'(x) = \frac{-3x^2 \sin(x^3+2)}{\cos(x^3+2)}$

We first simplify: $h(x) = x^2 + \exp(\cos(x) + x^2 + 1)$. Then

$h'(x) = 2x + (-\sin(x) + 2x)\exp(\cos(x) + x^2 + 1)$.

Some statistics:

Score histogram:

≤ 4	5-10	11-15	16-20
2	1	2	0

Average score: 6.4

Lowest score: -2

Highest score: 14

Percentage scored per exercise:

1	2	3	4	5
80	10	15	20	40