Lehrstuhl A für Mathematik
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Calculus and Linear Algebra, Worksheet 5

to be discussed on 13 November 2008

Exercise 1.

Compute all possible products A_iA_j of the following matrices.

$$A_1 = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 4 & -1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 3 & 5 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 1 & 2 \\ 3 & 5 \\ 8 & 12 \\ 20 & 28 \end{pmatrix}.$$

Exercise 2.

Show that $B = A^{-1}$.

a) $A = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 4 & -3 \\ -5 & 4 \end{pmatrix}$;

b) $A = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & -2 & -3 \end{pmatrix}$.

Exercise 3.

a) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $a, b, c, d \in \mathbb{R}$ and $ad - bc \neq 0$. Show that

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

b) Using this formula, compute the inverse of the matrix A as given in Exercise 2 a).

Exercise 4.

Let A be one of the following matrices.

i) $\begin{pmatrix} -1 & 4 & 0 \\ 2 & -7 & 4 \\ 1 & -2 & 4 \end{pmatrix}$ ii) $\begin{pmatrix} 2 & 6 & 10 \\ 5 & -1 & 8 \\ 4 & -4 & 3 \end{pmatrix}$ iii) $\begin{pmatrix} 2 & 2 & 2 & 2 \\ 0 & 1 & 0 & 1 \\ -2 & -1 & -1 & -1 \\ 2 & 4 & 3 & 4 \end{pmatrix}$ iv) $\begin{pmatrix} 1 & 1 & 3 & 2 \\ 2 & 2 & 0 & -1 \\ 4 & 0 & 3 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$

a) If possible, compute the inverse of A using the Gauss algorithm.

b) Is the system $Ax = b$ of linear equations uniquely solvable? If yes, compute the solution (for i), ii) let $b = (1,1,1)^T$, and for iii), iv) let $b = (1,1,1,1)^T$).

Exercise 5.

For which values $t \in \mathbb{R}$ is the matrix A_t invertible? Compute the inverse of A_1 and check your result.

a) $\begin{pmatrix} -1 & 4 & 0 \\ 2 & -7 & 0 \\ t & -2 & 4 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 4 & 0 \\ 2 & -7 & 4 \\ t & -2 & 4 \end{pmatrix}$ c) $\begin{pmatrix} 2 & 1 \\ t & t \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 & 0 & t \\ 0 & 2+t & 4 & 1 \\ 3 & 0 & 1 & 4t \\ 1 & 8+4t & 16 & 5+t \end{pmatrix}$

Exercise 6.

Compute the determinants of the following matrices.

a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ b) $\begin{pmatrix} 1+i & 2 \\ 2 & 2-i \end{pmatrix}$ c) $\frac{1}{2}\begin{pmatrix} 1 & 0 & 2 \\ i & i & 0 \\ 2 & -2 & -3 \end{pmatrix}$ d) $2\begin{pmatrix} 2 & -2 & 4 \\ 3 & 2 & 0 \\ 4 & 0 & 1 \end{pmatrix}$ e) $\frac{1}{2}\begin{pmatrix} 1 & 0 & 2 \\ i & i & 0 \\ 2 & -2 & -3 \end{pmatrix}$

f) $\begin{pmatrix} 1 & 0 & 9 & -3 \\ 3 & 4 & 1 & 3 \\ 0 & 8 & 0 & 1 \\ 2 & 1 & 3 & 10 \end{pmatrix}$ g) $\begin{pmatrix} 7 & 2 & 9 & 1 \\ 5 & 3 & 6 & 12 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & -1 & 6 \end{pmatrix}$ h) $\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Exercise 7.

Compute the solutions of the system $Ax = b$ of linear equations using Cramer's rule.

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 4 & 2 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 2 & 0 & -3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

c) $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & -1 & 0 & 2 \\ 4 & 2 & 3 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$