Lehrstuhl A für Mathematik
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Calculus and Linear Algebra, Worksheet 9

to be discussed on Thursday, 11 December 2008

Exercise 1.

Let f be the function defined by $f(x) =$

a) $x\sqrt{1+\sqrt{x}}$ b) $(2x^3+3x^2+6x-8)(2\sqrt{x}-6\sqrt[3]{x}+2\sqrt{x^3})$

c) $\dfrac{3x-6}{x^2-4x+5}$ d) $\sqrt[5]{(2x-5)^4}$ e) $\sqrt{\dfrac{1+x}{1-x}}$

Compute the domain of continuity of f. Where is f differentiable? Compute the derivative of f.

Exercise 2.

Let f be defined by $f(x) =$

a) $\sqrt{1+\sin x\cos x}$ b) $\sqrt{a^2+x^2}$ $(a \in \mathbb{R}\setminus\{0\})$ c) $\sqrt[5]{(5x-2)^4}$ d) $(x-1)\cdot|x-1|$

e) $\dfrac{x^2|x+1|}{|x-2|}$ f) $\dfrac{\sin x}{x(1-\cos x)}$ g) $\sqrt{\dfrac{2-x}{2+x}}$ h) $\sqrt{x+\sqrt[3]{x+\sqrt[4]{x}}}$

Determine the domain of f, the derivative f' of f and the domain of f'.

Exercise 3.

Where are the following functions continuous? Where are they differentiable? Compute the derivatives.

$$\text{a) } f(x) = \begin{cases} 2+2x, & x<-1 \\ x(x^2-1), & -1\leqslant x<2 \\ 11x-17, & 2\leqslant x\leqslant 3 \\ 16, & x>3 \end{cases} \qquad \text{b) } f(x) = \begin{cases} \dfrac{5}{2x}, & x<-1 \\ x^2-\dfrac{7}{2}, & -1\leqslant x\leqslant 2 \\ \dfrac{1}{x-1}, & x>2 \end{cases}$$

Exercise 4.

Determine the tangent of the curve $(x, f(x))$ in the point $(0, f(0))$.

a) $f(x) = \sqrt{(x+2)^3}$ b) $f(x) = \dfrac{2x}{1+x^2}$ c) $f(x) = \sqrt[3]{(x+1)^2}$

Exercise 5.

Let the functions $f, g, h : \mathbb{R} \to \mathbb{R}$ be defined by $h(x) = \max\{0, 1 - |x|\}$, $f(x) = (x+1)h(x)$ und $g(x) = xh(x)$.

a) Show that f, g and h are continuous;

b) Determine $f'(-1)$. Show that $f'(0)$ does not exist;

c) Determine $g'(0)$. Show that $g'(1)$ does not exist;

d) Sketch the graphs of f, g and h.

Exercise 6.

Find the Taylor polynomial $T_{n,x_0}(y)$ of the following functions and estimate the error R_{n,x_0} in the region defined by the given condition.

a) $f(x) = \cos(\frac{\pi}{4} \sin x)$, $|x + \frac{\pi}{2}| \leqslant 10^{-1}$, $x_0 = -\frac{\pi}{2}$, $n = 2$;

b) $f(x) = \sin^2(x)$, $|x - \frac{\pi}{2}| \leqslant r$, where $r > 0$, $x_0 = \frac{\pi}{2}$, $n = 2$;

c) $f(x) = \dfrac{1}{x+2}$, $|x| \leq \frac{1}{10}$, $x_0 = 0$, $n = 3$.

Exercise 7.

Using the mean value theorem, show that the following inequalities hold.

a) $\frac{1}{2}x \leqslant \arctan x \leqslant x$ for $x \in [0, 1]$ b) $x \leqslant \tan x \leqslant 2x$ for $x \in [0, \frac{\pi}{4}]$