Lehrstuhl A für Mathematik
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Calculus and Linear Algebra, Worksheet 13

to be discussed on Thursday, 22 January 2009

Exercise 1.

Let $a, b \in R$ such that $a, b \neq 0$. Solve the following integrals using integration by parts and find out the admissible parameter values $\alpha, \beta \in \mathbb{R}$ for the boundaries of integration. What happens in the exercises f), g) and h) in the special case $\alpha = -\pi$, $\beta = \pi$ and $a, b \in \mathbb{N}$?

a) $\int_\alpha^\beta x \arctan x \, dx$ b) $\int_\alpha^\beta x \ln x \, dx$ c) $\int_\alpha^\beta e^{at} \cos(bt) \, dt$

d) $\int_\alpha^\beta \ln^2 x \, dx$ e) $\int_\alpha^\beta \frac{xe^x}{(1+x)^2} \, dx$ f) $\int_\alpha^\beta \cos(ax)\cos(bx) \, dx$

g) $\int_\alpha^\beta \sin(ax)\sin(bx) \, dx$ h) $\int_\alpha^\beta \cos(ax)\sin(bx) \, dx$

Exercise 2.

Find a primitive $F(x)$ for the given function $f(y)$ using an adequate substitution.

a) $f(y) = \frac{1}{y\sqrt{y-9}}$ b) $f(y) = \frac{1}{2e^{6y} + 8e^{-6y}}$ c) $f(y) = \ln(1 + \frac{y}{2})$

d) $f(y) = \cos\sqrt{y}$ e) $f(y) = \frac{\ln^2 y}{y^2}$ f) $f(y) = \frac{\arctan(\ln y)}{y}$

Exercise 3.

Try to solve the following integrals with an *educated guess* or with an adequate substitution (assuming that $\alpha, \beta \in \mathbb{R}$ are appropriate boundaries).

a) $\int_\alpha^\beta \frac{\arcsin 2t}{\sqrt{1-4t^2}} \, dt$ b) $\int_\alpha^\beta xe^{a+bx^2} \, dx \quad (b \neq 0)$ c) $\int_\alpha^\beta \frac{x}{x^2+1} \, dx$

d) $\int_\alpha^\beta \frac{\sqrt{\ln x}}{x} \, dx$ e) $\int_\alpha^\beta \frac{\arctan^3 x}{1+x^2} \, dx$ f) $\int_\alpha^\beta x\sqrt{1-x^2} \, dx$

Exercise 4.

Solve the following integrals using partial fraction decomposition and calculate adequate integration boundaries $\alpha, \beta \in \mathbb{R}$.

a) $\displaystyle\int_\alpha^\beta \frac{3}{x^3-1}\,dx$ b) $\displaystyle\int_\alpha^\beta \frac{36}{(x-4)(x+2)^2}\,dx$ c) $\displaystyle\int_\alpha^\beta \frac{x^3-x^2+2x-5}{x^2-x-2}\,dx$

d) $\displaystyle\int_\alpha^\beta \frac{x^2-3x+2}{(2x+5)(x^2-1)}\,dx$ e) $\displaystyle\int_\alpha^\beta \frac{2x^2-x+8}{(x-2)(x^2+x+1)}\,dx$ f) $\displaystyle\int_\alpha^\beta \frac{3x^3-2x+5}{x^2-4x+3}\,dx$

g) $\displaystyle\int_\alpha^\beta \frac{x^3+x^2+1}{(x+1)^2(x^2+1)}\,dx$ h) $\displaystyle\int_\alpha^\beta \frac{x^5+1}{x^2+x^3}\,dx$ i) $\displaystyle\int_\alpha^\beta \frac{6x^2+x+3}{3x+5}\,dx$

Exercise 5.

Solve the following integrals ($\alpha, \beta \in \mathbb{R}$, if nothing more is said).

a) $\displaystyle\int_\alpha^\beta \frac{dy}{y\ln y}$ $(0<\alpha,\beta<1 \text{ or } \alpha,\beta>1)$

b) $\displaystyle\int_\alpha^\beta (x^2+1)^{21}x^3\,dx$

c) $\displaystyle\int_\alpha^\beta \frac{y}{\cos^2 y}\,dy$ $(\pi(k-\frac{1}{2})<\alpha,\beta<\pi(k+\frac{1}{2}),\ k\in\mathbb{Z})$

d) $\displaystyle\int_\alpha^\beta \frac{t}{\sqrt{t^2+1}}\,dt$

e) $\displaystyle\int_\alpha^\beta \tan^2 t\,dt$ $(\pi(k-\frac{1}{2})<\alpha,\beta<\pi(k+\frac{1}{2}),\ k\in\mathbb{Z})$

f) $\displaystyle\int_\alpha^\beta \arctan\sqrt{y}\,dy$ $(\alpha,\beta\geq 0)$

g) $\displaystyle\int_\alpha^\beta (\cos x)\sqrt{\sin x}\,dx$ $(2k\pi \leqslant \alpha,\beta \leqslant (2k+1)\pi, k\in\mathbb{Z})$

h) $\displaystyle\int_\alpha^\beta x^2e^{x/2}\,dx$

j) $\displaystyle\int_\alpha^\beta \frac{dy}{\sqrt{y}+\sqrt{y+1}}$ $(\alpha,\beta\geqslant 0)$

k) $\displaystyle\int_\alpha^\beta 3^y\,dy$