Lehrstuhl A für Mathematik
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Calculus and Linear Algebra, Worksheet 14

to be discussed on Thursday, 29 January 2009

Exercise 1.

Do the following improper integrals exist?

a) $\displaystyle\int_0^1 \frac{2}{\sqrt{1-x^2}}\,dx$ b) $\displaystyle\int_0^\infty \frac{\arctan x}{x^{3/2}}\,dx$ c) $\displaystyle\int_0^{\pi/2} \tan x\,dx$

d) $\displaystyle\int_0^1 \frac{\sin x}{x^2}\,dx$ e) $\displaystyle\int_{-\infty}^\infty e^{-2x}\,dx$ f) $\displaystyle\int_{-\infty}^\infty e^{-2|x|}\,dx$

g) $\displaystyle\int_1^\infty \frac{\cos x \sin x}{x^2+5x+1}\,dx$ h) $\displaystyle\int_0^\infty \frac{e^x}{1+e^{2x}}\,dx$ i) $\displaystyle\int_0^1 \ln(x)\,dx$

Exercise 2.

Compute the following integrals.

a) $\displaystyle\iint_I 2x+3y\;d(x,y),\; I=[0,2]\times[3,4]$

b) $\displaystyle\iint_I xy+y^2\;d(x,y),\; I=[-2,1]\times[0,2]$

c) $\displaystyle\iint_I e^{x+y}\;d(x,y),\; I=[1,2]\times[1,2]$

d) $\displaystyle\iint_I \sin(x+y)\;d(x,y),\; I=[0,\tfrac{\pi}{2}]\times[0,\tfrac{\pi}{2}]$

e) $\displaystyle\iiint_I \frac{2z}{(x+y)^2}\;d(x,y,z),\; I=[1,2]\times[2,3]\times[0,2]$

f) $\displaystyle\iiint_I \frac{x^2z^3}{1+y^2}\;d(x,y,z),\; I=[0,1]\times[0,1]\times[0,1]$

Exercise 3.

Compute the area integral $\iint\limits_G f(x,y)\,d(x,y)$ and sketch G.

a) $f(x,y) = 2e^{-x^2}$, $G = \{(x,y) \in \mathbb{R}^2 \,|\, 0 \leq x \leq 1,\ 0 \leq y \leq x\}$
b) $f(x,y) = x^2 + y^2$, $G = \{(x,y) \in \mathbb{R}^2 \,:\, x, y \geqslant 0,\ 0 \leqslant x + y \leqslant 1\}$
c) $f(x,y) = xy$, $G = \{(x,y) \in \mathbb{R}^2 \,:\, x^2 \leqslant y \leqslant x\}$
d) $f(x,y) = \sqrt{x}y$, $G = \{(x,y) \in \mathbb{R}^2 \,:\, x \geq 0,\ x^2 \leqslant y \leqslant \sqrt{x}\}$
e) $f(x,y) = 8e^5 x e^y$, $G = \{(x,y) \in \mathbb{R}^2 \,:\, x \geq 0,\ x^2 - 5 \leq y \leq -4x^2\}$
f) $f(x,y) = \pi x$, $G = \{(x,y) \in \mathbb{R}^2 \,:\, x \in [0,1],\ 0 \geq y \geq -\cos(\frac{\pi}{2}x^2)\}$

Exercise 4.

a) Compute the area of the ellipse $E(a,b)$, that is defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leqslant 1 \ \ (a, b > 0)$.

b) Compute the integral $\iint\limits_{E(a,b)} |x|\, d(x,y)$.

Hint: Partition the integral using the domains $x \geq 0$ and $x \leq 0$.