Lehrstuhl A für Mathematik
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Aachen, den 22.11.2001

5. Übung zu Zahlbereichserweiterungen

(Abgabe: Donnerstag, 29.11.2001, vor der Übung oder
bis 10 Uhr im Übungskasten vor dem Sekretariat des Lehrstuhls)

Achtung: Ab sofort findet die Vorlesung am Montag im Hörsaal III statt!

Aufgabe 1: Zeigen Sie, dass die auf $\mathbb{N}_0 \times \mathbb{N}_0$ durch

$$(n, m) \sim (k, j) \quad :\Leftrightarrow \quad n + j = m + k$$

definierte Relation $\sim$ eine Äquivalenzrelation ist. $\boxed{3}$

Aufgabe 2: Zeigen Sie, dass die in der Vorlesung eingeführte Multiplikation $\odot$ sowie die Kleiner-oder-Gleich-Relation $\circledleq$ auf $\mathbb{Z}$ unabhängig von den gewählten Repräsentanten sind. $\boxed{4}$

Aufgabe 3: (vgl. Lemma 3.8 der Vorlesung)
In einem Ring $(R, +, \cdot)$ gelten für $a, b, c \in R$ die folgenden Regeln:

a) Aus $a + c = b + c$ folgt $a = b$. $\boxed{1}$

b) Die Gleichung $a + x = b$ hat genau eine Lösung $x \in R$, nämlich $x = b + (-a)$. $\boxed{1}$

c) $-(-a) = a$, $-(a + b) = (-a) + (-b)$, $-0 = 0$. $\boxed{1}$

d) $a \cdot 0 = 0 \cdot a = 0$. $\boxed{1}$

e) $-(a \cdot b) = (-a) \cdot b = a \cdot (-b)$. $\boxed{1}$

f) $(-a) \cdot (-b) = a \cdot b$. $\boxed{1}$

Aufgabe 4: Beweisen Sie: $(\mathbb{Z}, +, \cdot)$ ist ein kommutativer, unitärer Ring. $\boxed{8}$

Aufgabe 5: (vgl. Lemma 3.14 der Vorlesung)
Sei $(R, <)$ ein angeordneter, unitärer Ring. Zeigen Sie, daß für alle a, b, c, $d \in R$ gilt:

a) $a < b$ ist äquivalent zu $-b < -a$. $\boxed{1}$

b) Aus $a < b$ und $c < d$ folgt $a + c < b + d$. $\boxed{1}$

c) Aus $0 \leq a < b$ und $0 \leq c < d$ folgt $ac < bd$. $\boxed{2}$

d) Aus $a < b$ und $c < 0$ folgt $ac > bc$ und $ca > cb$. $\boxed{2}$

e) Es gilt $1 > 0$ und $a^2 > 0$ für alle $a \in R$, $a \neq 0$. $\boxed{3}$

f) Sei $0 < a < b$, so daß $a^{-1} \in R$ und $b^{-1} \in R$ existieren. Dann gilt $0 < b^{-1} < a^{-1}$. $\boxed{3}$

$\boxed{\boxed{33}}$

Johann Peter Gustav Lejeune Dirichlet[1]

Born: 13 Feb 1805 in Düren, French Empire (now Germany)
Died: 5 May 1859 in Göttingen, Hanover (now Germany)

Dirichlet taught at the University of Breslau in 1827 and the University of Berlin from 1828 to 1855. He then succeeded to Gauss's chair at Göttingen.
He proved in 1826 that in any arithmetic progression with first term coprime to the difference there are infinitely many primes. This had been conjectured by Gauss. His work on units in algebraic number theory *Vorlesungen über Zahlentheorie* (published 1863) contains important work on ideals. He also proposed in 1837 the modern definition of a function.

If a variable y is so related to a variable x that whenever a numerical value is assigned to x, there is a rule according to which a unique value of y is determined, then y is said to be a function of the independent variable x.

In mechanics he investigated the equilibrium of systems and potential theory. This led him to the Dirichlet problem concerning harmonic functions with given boundary conditions. Dirichlet is best known for his papers on conditions for the convergence of trigonometric series and the use of the series to represent arbitrary functions. These series had been used previously by Fourier in solving differential equations. Dirichlet's work is published in Crelle's Journal in 1828. Earlier work by Poisson on the convergence of Fourier series was shown to be nonrigorous by Cauchy. Cauchy's work itself was shown to be in error by Dirichlet who wrote of Cauchy's paper

The author of this work himself admits that his proof is defective for certain functions for which the convergence is, however, incontestable.

Because of this work Dirichlet is considered the founder of the theory of Fourier series.
At age 45 he was described by Thomas Hirst as follows:

He is a rather tall, lanky-looking man, with moustache and beard about to turn grey with a somewhat harsh voice and rather deaf. He was unwashed, with his cup of coffee and cigar. One of his failings is forgetting time, he pulls his watch out, finds it past three, and runs out without even finishing the sentence.

Riemann was a student of Dirichlet. Gustav L Dirichlet was elected to the Royal Society of London in 1855. There is a *Crater Dirichlet* on the moon.

[1] Aus: 'The MacTutor History of Mathematics archive' der University of St Andrews, Scotland.