On Hilbert modular forms for $\mathbb{Q}(\sqrt{5})$, $\mathbb{Q}(\sqrt{13})$ and $\mathbb{Q}(\sqrt{17})$

Errata

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I do know of the following mistakes:

- page 44, $\mathfrak{u} = \mathfrak{d} = \mathfrak{o}$ is wrong. Better is $\mathfrak{u} = \mathfrak{o}$ (and $\mathfrak{d} = (\sqrt{p})$).
- page 58, proof of Remark 2.5.11: Replace β by $f^{(p)}$ and α by f, so

  The transformation property comes from

$$\begin{aligned} f^{(p)}\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\langle\tau\rangle\right) &= f\left(p\frac{a\tau+b}{c\tau+d}\right) \\ &= f\left(\frac{ap\tau+pb}{(c/p)(p\tau)+d}\right) \\ &= f\left(\begin{pmatrix} a & pb \\ c/p & d \end{pmatrix}\langle p\tau\rangle\right) \\ &= \nu\left(\begin{pmatrix} a & pb \\ c/p & d \end{pmatrix}\right)(c\tau+d)^k f^{(p)}(\tau)\ . \end{aligned}$$

- page 69, Definition 2.5.37: The n in $\chi_p(n) \neq -1$ has to be an m. So the sentence is: In case $p \in \{5, 13, 17\}$, for all $m \in \mathbb{N}$ with $\chi_p(m) \neq -1$ we write ...
- page 76, proof of Lemma 3.2.2, first two lines: Define $\lambda = \lambda_1 + \lambda_2\frac{\sqrt{p}}{p}$ and not $\lambda = \lambda_1 + \lambda_2\sqrt{p}$. In line six one should then write "‘It is $\mathrm{N}(\lambda) = -m/p$"‘ instead of "‘Let $\mathrm{N}(\lambda_1 + \lambda_2\sqrt{p}/p) = -m/p$"’.
- page 77, proof of Lemma 3.2.2, $m < 0$: Replace a by λ_1.
- page 80, proof of Lemma 3.3.2: Here we use that W is open, but give no proof.

  Remark 3.2.2. *Each Weyl chamber W is an open set*

  Proof. Consider $m \in \mathbb{N}$ ($m > 0$). Since there is a finite set $\mathcal{J}$ as in Lemma 3.2.2 on page 75 with

$$\mathcal{I} = \left\{\lambda \in \frac{\mathfrak{o}}{\sqrt{p}} \mid \mathrm{N}(\lambda) = -\frac{m}{p}\right\} = \left\{\pm\lambda\varepsilon_0^{2k} \mid k \in \mathbb{Z}, \lambda \in \mathcal{J}\right\},$$

the set $\mathcal{I}$ is closed in $\mathbb{R}^*$, so also $\bigcup_{\lambda\in\mathcal{I}}\left\{\tau\in\mathbb{H}^2 \mid \operatorname{Im}(y_1)=\frac{\lambda}{\overline{\lambda}}\operatorname{Im}(y_2)\right\}$ is closed in $\mathbb{H}^2$. As a consequence, each component of the complement ist open. $\square$

- page 81, Lemma 3.3.4 should be stated more clearly:

  Lemma 3.3.4 (Calculation of $R(W,n)$). *Let p be an odd prime, let $m=-n$ be a natural number and $\tau\in W$ for some Weyl chamber W. Then $R(W,n)$ can be calculated by*

  For every element λ in a set of representatives of $R(-m)$ modulo multiplication with ε_0^2 do

  - *Multiply λ with ε_0^2 (and denote the result again by λ) until $\lambda y_1+\overline{\lambda}y_2>0$ for the imaginary part y of τ.*
  - *Multiply λ with ε_0^{-2}, until $\lambda y_1+\overline{\lambda}y_2<0$.*

  The resulting $M(\lambda)$ is an element of $R(W,n)$ and this procedure gives all of its elements when applied to all λ in $R(-m)/\varepsilon_0^2$.

- page 112, 5.1:
  - equation (5.1): In case n is even and $n<p$ we define

  $$\tilde{f}_n=f_{n-\frac{p-1}{\gcd(p-1,24)}}\cdot\tilde{H},\qquad \text{if } \frac{p-1}{\gcd(p-1,24)}<n<p \text{ and } n \text{ is even.}$$

  - Alternatively we can write $\tilde{f}_n=f_1^{n-2}f_2$ for even $n<p$.

- page 113, 5.1, Remark 5.1.2: A sign is false.

  The correct definition of $f_{(2)}$ is $f_{(2)}=q^{+l}\left(\sum_{n=0}^{N}\dots\right)$.

- page 113, 5.1.1: The equation for $\tilde{H}$ misses the factor q^{-2} in the Fourier expansion. The correct equation is

  $$\tilde{H}=\eta^6/(\eta^{(5)})^6=H^{(1)}/H^{(q)}=q^{-1}-6+9q+10q^2-30q^3+6q^4-25q^5+O\left(q^6\right),$$

- page 114, 5.1.2: The equation for $\tilde{H}$ misses the factor q^{-2} in the Fourier expansion. The correct equation is

  $$\tilde{H}=\eta^2/(\eta^{(13)})^2=q^{-1}-2-q+2q^2+q^3+2q^4-2q^5-2q^7+O\left(q^8\right),$$

- page 116, 5.1.3: The equation for $\tilde{H}$ misses the factor q^{-4} in the Fourier expansion. The correct equation is

  $$\tilde{H}=\eta^3/(\eta^{(17)})^3=q^{-2}-3q^{-1}+5q-7q^4+9q^8-11q^{13}+3q^{15}-9q^{16}+O\left(q^{18}\right),$$

- page 117, 5.1.3: Here obviously $f_{13} \neq f_1^13 - \frac{13}{2}f_{12} + \dots$ but

$$f_{13} = f_1^{13} - \frac{13}{2}f_{12} - \frac{13}{2}f_{11} + \frac{117}{4}f_{10} + \frac{221}{16}f_9 - \frac{6279}{32}f_8 - \frac{1157}{8}f_7 + \frac{14989}{32}f_6 + \frac{40105}{256}f_5 - \frac{653003}{512}f_4 - \frac{661193}{512}f_3 + \frac{927173}{1024}f_2 + \frac{12277915}{4096}f_1$$

- page 125, Step 1: the last part of the sentence is misleading formulated and wrong. It schould read: [...] and in the case that $\mathrm{S}(\rho_W)$ is negative, rewrite $k := k - \mathrm{S}(\rho_W)$.

- page 126, 5.3: Since $\nu \operatorname{Im}(\tau_1) + \overline{\nu} \operatorname{Im}(\tau_2) > 0$ by $(W, \nu) > 0$ and

  $$|\mathbf{e}(\nu\tau_1 + \overline{\nu}\tau_2)| = e^{-2\pi(\nu \operatorname{Im}(\tau_1) + \overline{\nu} \operatorname{Im}(\tau_2))} < 1,$$

  the geometric series converges.

- page 141, Lemma 7.10: The given values of the field automorphisms are wrong, for example $\pi_2(\sqrt{p}) = -\sqrt{p} \neq -1$. We better write

  [...] we fix the fields automorphisms π_1, π_2, π_3 and π_4 in the following way:

	$\mathrm{sign}(\pi_1)$	$\mathrm{sign}(\pi_2)$	$\mathrm{sign}(\pi_3)$	$\mathrm{sign}(\pi_4)$
$\sqrt{p}$	$+$	$-$	$+$	$-$
$\sqrt{q}$	$+$	$+$	$-$	$-$
$\sqrt{pq}$	$+$	$-$	$-$	$+$

  then the functions [...]