

## Content

This is an example of a MAPLE 7 worksheet for normalization, finding invariants and reduction of the following system:

$Dx = Mx + f_2(x) + f_3(x) + \text{higher order terms}$  , where:

$$M := \begin{bmatrix} \alpha_1 & \alpha_2 & 0 \\ \alpha_3 & -\alpha_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_2 := [\beta_1 x_1^2 + \beta_2 x_1 x_2 + \beta_3 x_2^2 + \mu_1 x_3 x_1 + \mu_2 x_2 x_3, \beta_4 x_1^2 + \beta_5 x_1 x_2 + \beta_6 x_2^2 + \mu_3 x_3 x_1 + \mu_4 x_2 x_3, 0]$$

$$f_3 := [\gamma_1 x_1^3 + \gamma_2 x_1^2 x_2 + \gamma_3 x_1 x_2^2 + \gamma_4 x_2^3 + \nu_1 x_1^2 x_3 + \nu_2 x_1 x_2 x_3 + \nu_3 x_2^2 x_3 + \rho_1 x_1 x_3^2 + \rho_2 x_2 x_3^2,$$

$$\gamma_5 x_1^3 + \gamma_6 x_1^2 x_2 + \gamma_7 x_1 x_2^2 + \gamma_8 x_2^3 + \nu_4 x_1^2 x_3 + \nu_5 x_1 x_2 x_3 + \nu_6 x_2^2 x_3 + \rho_3 x_1 x_3^2 + \rho_4 x_2 x_3^2, 0]$$

## Disclaimer

While our testing, as well as computations of examples, indicate correctness and reliability of the programs, the authors cannot guarantee the correctness of any routine.

Program developed by S. Mayer (email: mayer@mathA.rwth-aachen.de)

© S. Mayer, Aachen, 2003

The programs may be used for any non-commercial purpose by individuals and scientific organizations.

## Initialization

```
with(linalg):  
Warning, the protected names norm and trace have been redefined and  
unprotected
```

```
The Message: "Warning, the protected names norm and trace have been redefined and  
unprotected" will not inflict the following calculations. It derives from using the linalg package  
instead of the LinearAlgebra package, for the "linalg package is useful for doing computations  
in abstract linear algebra" (Maple glossary). This allows parameter dependent calculations.
```

## Primitive operations

## Procedures

## Input

The vector field  $v$  is given in the form  $v(x)=Mx+f_2[x]+...$  where  $M$  is an  $n \times n$  matrix and the  $f_j$  are homogeneous polynomials  $C^n \rightarrow C^n$  of degree  $j$ . The following routines are capable of working with parameters, so you might want to apply them for parameter dependent vector fields. Note that maple assumes non-degeneracies of all parameters.

```
Enter the linearization M of the vector field. The following calculations assume that M is  
semisimple.
```

```
> M:=matrix(3,3,[alpha[1],alpha[2],0,alpha[3],-alpha[1],0,0,0,0  
]);
```

$$M := \begin{bmatrix} \alpha_1 & \alpha_2 & 0 \\ \alpha_3 & -\alpha_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let targetspace be the desired degree of the normal form. targetspace then is also the degree of the Taylor expansion of the vector field.

```
> targetspace:=3;
```

```
targetspace := 3
```

Just evaluate the following lines.

```
> f:=vector(targetspace,0);
```

```
> dim:=rowdim(M);
```

```
dim := 3
```

```
> for i from 1 to targetspace do
```

```
>   f[i]:=vector(dim,0);
```

```
> od:
```

```
> f[1]:=Matrix2Polynom(M);
```

```
f1 := [α1 x1 + α2 x2, α3 x1 - α1 x2, 0]
```

Enter the higher order terms of the Taylor expansion of the vector field: f[2]:=vector(...);

```
f[3]:=... f[targetspace]:=vector(...);
```

```
> f[2]:=vector([beta[1]*x[1]^2+beta[2]*x[1]*x[2]+beta[3]*x[2]^2
+mu[1]*x[3]*x[1]+mu[2]*x[2]*x[3]
,beta[4]*x[1]^2+beta[5]*x[1]*x[2]+beta[6]*x[2]^2+mu[3]*x[3]*x
[1]+mu[4]*x[2]*x[3],0]);
```

```
f[3]:=vector([gamma[1]*x[1]^3+gamma[2]*x[1]^2*x[2]+gamma[3]*x
[1]*x[2]^2+gamma[4]*x[2]^3+nu[1]*x[1]^2*x[3]+nu[2]*x[1]*x[2]*
x[3]+nu[3]*x[2]^2*x[3]+rho[1]*x[1]*x[3]^2+rho[2]*x[2]*x[3]^2
,gamma[5]*x[1]^3+gamma[6]*x[1]^2*x[2]+gamma[7]*x[1]*x[2]^2+ga
mma[8]*x[2]^3+nu[4]*x[1]^2*x[3]+nu[5]*x[1]*x[2]*x[3]+nu[6]*x[
2]^2*x[3]+rho[3]*x[1]*x[3]^2+rho[4]*x[2]*x[3]^2,0]);
```

```
f2 :=
```

```
[β1 x12 + β2 x1 x2 + β3 x22 + μ1 x3 x1 + μ2 x2 x3, β4 x12 + β5 x1 x2 + β6 x22 + μ3 x3 x1 + μ4 x2 x3, 0]
```

```
f3 := [
```

```
γ1 x13 + γ2 x12 x2 + γ3 x1 x22 + γ4 x23 + ν1 x12 x3 + ν2 x1 x2 x3 + ν3 x22 x3 + ρ1 x1 x32 + ρ2 x2 x32,
```

```
γ5 x13 + γ6 x12 x2 + γ7 x1 x22 + γ8 x23 + ν4 x12 x3 + ν5 x1 x2 x3 + ν6 x22 x3 + ρ3 x1 x32 + ρ4 x2 x32, 0
```

```
]
```

Enter the maximal degree of calculated invariant polynomials.

```
> invariantsdegree:=2;
```

```
invariantsdegree := 2
```

## Automatic initialization

```
> dim:=rowdim(M);
```

```
> minimalpoly:=minpoly(M,tau);
```

```
minimalpoly := (-α12 - α2 α3) τ + τ3
```

```
> sigmaset:=symmetriceigenvalues(minimalpoly,dim);
```

## Determining annihilating polynomials

```
[ Annihilating polynomials for the action of ad M on P_r: (Remember that x -> p(-x) annihilates
the action of ad M on P_0 if p annihilates M.)
[ > liebracketannihilator:=AnnihilatingPolynomials(sigmaset,subs(
tau=-tau,minimalpoly),minimalpoly,dim,targetspace+1):
"Finished with step ", 1, "."
"Finished with step ", 2, "."
"Finished with step ", 3, "."
"Finished with step ", 4, "."
[ Now liebracketannihilator[j] annihilates the action of ad M on P_{j-1}.
[ > invpoly:=AnnihilatingPolynomials(sigmaset,minimalpoly,minimal
poly,dim,invariantsdegree):
"Finished with step ", 1, "."
"Finished with step ", 2, "."
[ Now invpoly[j] annihilates the action of L_M on S_j.
```

### Output

```
> for j from 1 to targetspace+1 do
>   print("Annihilating polynomial for ad M on homogeneous
polynomials of degree",j-1);
>   print(liebracketannihilator[j]);
> od;
"Annihilating polynomial for ad M on homogeneous polynomials of degree", 0

$$\tau(-\tau^2 + \alpha_1^2 + \alpha_2 \alpha_3)$$

"Annihilating polynomial for ad M on homogeneous polynomials of degree", 1

$$(-4 \alpha_2 \alpha_3 + \tau^2 - 4 \alpha_1^2) \tau(\tau^2 - \alpha_1^2 - \alpha_2 \alpha_3)$$

"Annihilating polynomial for ad M on homogeneous polynomials of degree", 2

$$(-4 \alpha_2 \alpha_3 + \tau^2 - 4 \alpha_1^2) \tau(\tau^2 - \alpha_1^2 - \alpha_2 \alpha_3)(-9 \alpha_2 \alpha_3 - 9 \alpha_1^2 + \tau^2)$$

"Annihilating polynomial for ad M on homogeneous polynomials of degree", 3

$$(-16 \alpha_2 \alpha_3 + \tau^2 - 16 \alpha_1^2)(-4 \alpha_2 \alpha_3 + \tau^2 - 4 \alpha_1^2) \tau(\tau^2 - \alpha_1^2 - \alpha_2 \alpha_3)$$


$$(-9 \alpha_2 \alpha_3 - 9 \alpha_1^2 + \tau^2)$$

> print("Annihilating polynomial for L_M on homogeneous
polynomials of degree",0);print(1);
for j from 1 to invariantsdegree do
>   print("Annihilating polynomial for L_M on homogeneous
polynomials of degree",j);
>   print(invpoly[j]);
> od;
"Annihilating polynomial for L_M on homogeneous polynomials of degree", 0
1
"Annihilating polynomial for L_M on homogeneous polynomials of degree", 1

$$\tau(-\tau^2 + \alpha_1^2 + \alpha_2 \alpha_3)$$

"Annihilating polynomial for L_M on homogeneous polynomials of degree", 2

$$(-4 \alpha_2 \alpha_3 + \tau^2 - 4 \alpha_1^2) \tau(\tau^2 - \alpha_1^2 - \alpha_2 \alpha_3)$$

```

## Higher-order normalization

```

[ > dgl:=matrix(targetspace,targetspace): #the differential
[ equation in the various transformed states
[ > Trafo:=vector(targetspace):Trafo[1]:=vector(dim,0): #the
[ transformation is exp(Trafo).
[ > for i from 1 to targetspace do
[ >   dgl[1,i]:=eval(f[i]);
[ > od:
[ > for i from 2 to targetspace do
[   #print("starting with
[   ",collect(expand(liebracketannihilator[i+1]),tau));
[ >
[   erg:=semisimplecalculation(M,collect(expand(liebracketannihil
[ ator[i+1]),tau),f[i]);
[   #print("transformation and transformed part in normal
[ form calculated for one degree higher");
[ >   for j from 1 to i-1 do
[ >     dgl[i,j]:=eval(dgl[i-1,j]);
[ >   od;
[   #print("some terms are copied (degree < r)");
[ >   Trafo[i]:=eval(erg[1]);
[ >   dgl[i,i]:=eval(erg[2]);
[   #print("some terms are chosen as it was calculated
[ before");
[ >   for j from i+1 to targetspace do
[ >     dgl[i,j]:=eval(dgl[i-1,j]);
[ >     k:=1;
[ >     while j >= k*(i-1)+1 do
[ >       hlp:=scalarmul(dgl[i-1,j-k*(i-1)],1/k!);
[ >       for l from 1 to k do
[ >         hlp:=lie(erg[1],hlp);
[ >       od;
[ >       dgl[i,j]:=matadd(dgl[i,j],hlp);
[ >       k:=k+1;
[ >     od;
[ >     #print("transformation of higher terms up to degree
[ >     ",j," done in this step")
[ >   od;
[ >   for j from i to targetspace do
[ >     f[j]:=eval(dgl[i,j]);
[ >   od;
[   #print("transformation succesfull up to degree ",i);
[ > od:

```

The The previous lines compute Poincare-Dulac normal form up to degree targetspace. It is stored in the variable dgl. dgl is a field of dimension targetspace<sup>2</sup>:

dgl[i,1]+dgl[i,2]+...+dgl[i,targetspace] is the Poincare-Dulac-normal form up to degree i. The corresponding transformation is given by exp(ad(Trafo[i])) exp(ad(Trafo[i-1])) ... exp(ad(Trafo[2])). An output of the results is provided in the next steps:

#### — Output of the Poincare-Dulac normal form

```

[ > print("Poincare-Dulac normal form up to degree",
[   targetspace,":");
[   for i from 1 to targetspace do
[     print(dgl[targetspace,i]);
[ > od:

```

"Poincare-Dulac normal form up to degree", 3, ":"

$[\alpha_1 x_1 + \alpha_2 x_2, \alpha_3 x_1 - \alpha_1 x_2, 0]$

$$\begin{aligned}
& \left[ \frac{1}{2} x_3 (\alpha_2 \alpha_3 \mu_1 x_1 + 2 \mu_1 x_1 \alpha_1^2 + \alpha_2 \alpha_3 \mu_2 x_2 + \alpha_2 x_2 \mu_1 \alpha_1 - \alpha_2 x_2 \mu_4 \alpha_1 + \mu_3 x_2 \alpha_2^2 \right. \\
& \quad + \alpha_2 x_1 \mu_3 \alpha_1 + \alpha_3 x_1 \mu_2 \alpha_1 + \alpha_2 \alpha_3 x_1 \mu_4) / (\alpha_2 \alpha_3 + \alpha_1^2), \frac{1}{2} x_3 (\alpha_2 \alpha_3 \mu_3 x_1 \\
& \quad + \alpha_2 \alpha_3 \mu_4 x_2 + 2 \mu_4 x_2 \alpha_1^2 + \alpha_2 \alpha_3 x_2 \mu_1 - \alpha_2 \mu_3 \alpha_1 x_2 - \alpha_3 \mu_2 \alpha_1 x_2 + \alpha_3 \mu_1 \alpha_1 x_1 \\
& \quad \left. + x_1 \mu_2 \alpha_3^2 - \alpha_3 \mu_4 \alpha_1 x_1) / (\alpha_2 \alpha_3 + \alpha_1^2), 0 \right] \\
& \left[ -\frac{1}{24} (10 x_2^3 \beta_3^2 \alpha_3^3 \alpha_2^2 - 4 x_2^3 \alpha_2^3 \beta_6^2 \alpha_3^2 + 28 x_1^2 x_2 \beta_2 \beta_5 \alpha_1^3 \alpha_2 \alpha_3 \right. \\
& \quad - 28 x_1^2 x_2 \beta_1 \beta_2 \alpha_1^3 \alpha_2 \alpha_3 + 24 x_1^2 x_2 \beta_1 \beta_2 \alpha_1^5 - x_1^2 x_2 \beta_5^2 \alpha_3^2 \alpha_2^3 \\
& \quad - 16 x_1^3 \alpha_2^2 \beta_6 \alpha_3^2 \alpha_1 \beta_4 + 4 x_1^3 \alpha_2 \beta_6 \alpha_3 \alpha_1^3 \beta_4 + 17 x_1^2 x_2 \beta_5^2 \alpha_3 \alpha_2^2 \alpha_1^2 \\
& \quad - 9 \gamma_4 x_2^3 \alpha_2^2 \alpha_3^2 \alpha_1^2 + 11 x_1^3 \beta_1 \alpha_3^2 \beta_2 \alpha_2 \alpha_1^2 - 9 \gamma_4 x_2^3 \alpha_2^3 \alpha_3^3 \\
& \quad - 6 x_3^2 x_1 \alpha_2^2 \mu_4 \alpha_3 \mu_3 \alpha_1^2 + 6 x_3^2 x_1 \mu_1 \alpha_2^2 \mu_3 \alpha_3 \alpha_1^2 - 6 x_3^2 x_1 \mu_2 \alpha_2 \mu_4 \alpha_3^2 \alpha_1^2 \\
& \quad + 6 x_3^2 x_1 \mu_1 \mu_2 \alpha_3^2 \alpha_2 \alpha_1^2 - x_2^3 \beta_2^2 \alpha_3^2 \alpha_2^3 + 24 x_1^2 x_2 \beta_2^2 \alpha_1^4 \alpha_3 \\
& \quad + 46 x_1^2 x_2 \beta_2 \beta_3 \alpha_3^2 \alpha_1^3 - 5 x_1^2 x_2 \beta_2^2 \alpha_1^2 \alpha_3^2 \alpha_2 - 14 x_1^2 x_2 \beta_2 \beta_3 \alpha_3^3 \alpha_1 \alpha_2 \\
& \quad + x_1^2 x_2 \beta_2 \beta_4 \alpha_2^3 \alpha_3^2 - 8 x_1^2 x_2 \beta_2 \beta_5 \alpha_3^2 \alpha_2^2 \alpha_1 - 20 x_1^2 x_2 \beta_1 \alpha_2^2 \beta_6 \alpha_3^2 \alpha_1 \\
& \quad + 40 x_1^2 x_2 \beta_1 \alpha_2 \beta_6 \alpha_3 \alpha_1^3 + 20 x_1^2 x_2 \alpha_2^2 \beta_6 \alpha_3^2 \alpha_1 \beta_5 - 40 x_1^2 x_2 \alpha_2 \beta_6 \alpha_3 \alpha_1^3 \beta_5 \\
& \quad + 8 x_1^2 x_2 \beta_1 \alpha_3^2 \beta_2 \alpha_2^2 \alpha_1 - 16 x_1^2 x_2 \beta_3 \beta_4 \alpha_1^5 + 32 x_1^2 x_2 \beta_3 \beta_6 \alpha_3^3 \alpha_1 \alpha_2 \\
& \quad - 28 x_1^2 x_2 \beta_3 \beta_6 \alpha_3^2 \alpha_1^3 - 10 x_1^2 x_2 \beta_3^2 \alpha_3^4 \alpha_2 + 20 x_1^2 x_2 \beta_3^2 \alpha_3^3 \alpha_1^2 \\
& \quad - 12 x_1^2 x_2 \beta_3 \beta_4 \alpha_2^2 \alpha_3^2 \alpha_1 + 32 x_1^2 x_2 \beta_3 \beta_4 \alpha_2 \alpha_3 \alpha_1^3 - 8 x_1^2 x_2 \alpha_2 \beta_6 \alpha_1^4 \beta_4 \\
& \quad - 12 x_1^2 x_2 \alpha_2 \beta_5^2 \alpha_1^4 + 5 x_1^2 x_2 \beta_1 \beta_5 \alpha_3^2 \alpha_2^3 + 63 x_1 x_2^2 \beta_2 \beta_3 \alpha_3^2 \alpha_2 \alpha_1^2 \\
& \quad + 42 x_1^2 x_2 \alpha_2^2 \beta_6 \alpha_3 \beta_4 \alpha_1^2 - 15 x_1^2 x_2 \beta_2 \beta_4 \alpha_1^2 \alpha_2^2 \alpha_3 - 10 x_1^2 x_2 \alpha_2^3 \beta_6 \alpha_3^2 \beta_4 \\
& \quad + 44 x_1^2 x_2 \beta_2 \beta_4 \alpha_1^4 \alpha_2 + 3 x_1 x_2^2 \beta_2 \beta_3 \alpha_3^3 \alpha_2^2 - 24 x_1^2 x_2 \beta_2 \beta_6 \alpha_1^4 \alpha_3 \\
& \quad - 19 x_1^2 x_2 \beta_1 \alpha_2^2 \beta_5 \alpha_1^2 \alpha_3 + 31 x_1^2 x_2 \beta_2 \beta_6 \alpha_1^2 \alpha_3^2 \alpha_2 + 36 x_1^2 x_2 \beta_1 \alpha_2 \beta_5 \alpha_1^4 \\
& \quad + 2 x_1^2 x_2 \beta_1^2 \alpha_1^2 \alpha_2^2 \alpha_3 - 24 x_1^2 x_2 \beta_1^2 \alpha_1^4 \alpha_2 - 4 x_1^2 x_2 \beta_1^2 \alpha_3^2 \alpha_2^3 \\
& \quad - 12 x_1^3 \beta_1 \beta_2 \alpha_1^4 \alpha_3 + 26 x_1^3 \beta_1 \beta_4 \alpha_2^2 \alpha_3 \alpha_1^2 - 2 x_1^3 \beta_2 \beta_4 \alpha_2^2 \alpha_3^2 \alpha_1 \\
& \quad - 17 x_1^3 \beta_5 \alpha_3 \alpha_2^2 \beta_4 \alpha_1^2 + 6 x_1^3 \beta_1 \beta_4 \alpha_2^3 \alpha_3^2 + 10 x_1^2 x_2 \beta_4^2 \alpha_2^4 \alpha_3 \\
& \quad - 20 x_1^2 x_2 \beta_4^2 \alpha_2^3 \alpha_1^2 - 54 x_1 x_2^2 \beta_3 \beta_6 \alpha_1^2 \alpha_3^2 \alpha_2 + 24 x_1 x_2^2 \beta_2 \beta_5 \alpha_3 \alpha_2^2 \alpha_1^2 \\
& \quad + 36 x_1 x_2^2 \beta_1 \beta_3 \alpha_1^3 \alpha_2 \alpha_3 - 24 x_1 x_2^2 \beta_1^2 \alpha_2^2 \alpha_1^3 - 45 x_1 x_2^2 \alpha_2^2 \beta_6 \alpha_1^2 \beta_5 \alpha_3 \\
& \quad \left. - 3 \gamma_3 x_1 x_2^2 \alpha_2^3 \alpha_3^3 - 33 \gamma_3 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \alpha_1^2 - 3 \gamma_2 x_1^2 x_2 \alpha_2^3 \alpha_3^3 \right]
\end{aligned}$$

$$\begin{aligned}
& -3 \gamma_2 x_1^2 x_2^2 \alpha_2^2 \alpha_3^2 \alpha_1^2 - 12 \rho_2 x_2 x_3^2 \alpha_2^3 \alpha_3^3 - 24 \gamma_2 x_1^2 x_2 \alpha_1^4 \alpha_2 \alpha_3 \\
& - 30 \gamma_3 x_1 x_2^2 \alpha_1^4 \alpha_2 \alpha_3 + 12 x_1 x_2^2 \alpha_2 \beta_6 \alpha_1^4 \beta_5 + 36 x_1 x_2^2 \beta_4 \alpha_1 \alpha_2^3 \beta_6 \alpha_3 \\
& - 24 x_1 x_2^2 \beta_4 \alpha_1^3 \alpha_2^2 \beta_6 - 24 x_1 x_2^2 \alpha_2^2 \beta_5^2 \alpha_1^3 - 24 \rho_2 x_2 x_3^2 \alpha_1^2 \alpha_2^2 \alpha_3^2 \\
& - 12 \rho_2 x_2 x_3^2 \alpha_1^4 \alpha_2 \alpha_3 - 48 \rho_1 x_1 x_3^2 \alpha_2^2 \alpha_3^2 \alpha_1^2 - 60 \rho_1 x_1 x_3^2 \alpha_1^4 \alpha_2 \alpha_3 \\
& - 12 \rho_1 x_1 x_3^2 \alpha_2^3 \alpha_3^3 - 10 x_2^3 \beta_3 \alpha_3^2 \beta_1 \alpha_2^3 + x_2^3 \beta_3 \beta_5 \alpha_3^2 \alpha_2^3 - 12 x_1^3 \beta_2^2 \alpha_3^2 \alpha_1^3 \\
& + 5 x_2^3 \beta_2 \alpha_2^3 \beta_6 \alpha_3^2 - 20 x_2^3 \beta_3 \alpha_2^2 \beta_6 \alpha_3^2 \alpha_1 - 19 x_2^3 \beta_3 \beta_5 \alpha_1^2 \alpha_2^2 \alpha_3 \\
& + 8 x_2^3 \beta_2 \alpha_2^3 \beta_5 \alpha_1 \alpha_3 - 12 x_2^3 \beta_2 \alpha_2^2 \beta_5 \alpha_1^3 + 20 x_2^3 \beta_1 \alpha_2^3 \alpha_1 \beta_6 \alpha_3 \\
& - 14 x_2^3 \beta_2 \beta_1 \alpha_2^3 \alpha_1 \alpha_3 + 6 x_2^3 \beta_2 \beta_1 \alpha_2^2 \alpha_1^3 + 20 x_2^3 \beta_2 \beta_3 \alpha_3^2 \alpha_1 \alpha_2^2 \\
& - 8 x_2^3 \beta_3 \beta_4 \alpha_1^3 \alpha_2^2 - 27 \gamma_1 x_1^3 \alpha_1^2 \alpha_2^2 \alpha_3^2 - 18 \gamma_1 x_1^3 \alpha_1^4 \alpha_2 \alpha_3 \\
& + 10 x_2^3 \beta_1 \alpha_1^2 \alpha_2^2 \beta_3 \alpha_3 + 12 x_2^3 \beta_3 \beta_4 \alpha_1 \alpha_2^3 \alpha_3 - 15 x_2^3 \beta_2 \alpha_2^2 \beta_6 \alpha_1^2 \alpha_3 \\
& - 12 x_1^3 \beta_3 \alpha_3^2 \alpha_2 \beta_4 \alpha_1^2 - 8 x_1^3 \beta_2 \beta_6 \alpha_3^3 \alpha_1 \alpha_2 + 12 x_1^3 \beta_2 \beta_6 \alpha_3^2 \alpha_1^3 \\
& - 8 x_1^3 \beta_2 \beta_5 \alpha_3^2 \alpha_2 \alpha_1^2 + 8 x_1^3 \beta_3 \alpha_3 \alpha_1^4 \beta_4 + 12 x_1^3 \beta_2 \beta_5 \alpha_3 \alpha_1^4 \\
& - 14 x_2^3 \alpha_2^3 \beta_5 \alpha_1 \beta_6 \alpha_3 + 6 x_2^3 \alpha_2^2 \beta_5 \alpha_1^3 \beta_6 - 22 x_1^3 \beta_2 \beta_4 \alpha_2 \alpha_3 \alpha_1^3 \\
& + 3 x_1^3 \beta_5 \alpha_3^2 \alpha_2^3 \beta_4 - 24 \rho_1 x_1 x_3^2 \alpha_1^6 - 6 x_1 x_3^2 \alpha_3 \mu_2 \mu_4 \alpha_1^4 + 6 x_1 x_3^2 \alpha_3 \mu_1 \mu_2 \alpha_1^4 \\
& - 12 x_1 x_3^2 \alpha_2^2 \alpha_3^3 \rho_2 \alpha_1 - 24 x_1 x_3^2 \alpha_2^2 \alpha_3^2 \rho_4 \alpha_1^2 + 6 x_1 x_3^2 \alpha_2 \mu_3 \alpha_1^4 \mu_1 \\
& - 12 x_1 x_3^2 \alpha_3 \rho_2 \alpha_1^5 - 12 x_1 x_3^2 \alpha_2 \alpha_3 \rho_4 \alpha_1^4 - 24 x_1 x_3^2 \alpha_2 \alpha_3^2 \rho_2 \alpha_1^3 \\
& - 12 x_2 x_3^2 \alpha_2 \mu_2 \mu_3 \alpha_1^4 - 12 x_1 x_3^2 \alpha_2^3 \alpha_3^2 \rho_3 \alpha_1 + 24 x_2 x_3^2 \alpha_2^2 \alpha_3 \rho_4 \alpha_1^3 \\
& - 12 x_2 x_3^2 \alpha_2^2 \rho_3 \alpha_1^4 - 24 x_2 x_3^2 \alpha_2^2 \alpha_3 \rho_1 \alpha_1^3 - 12 x_2 x_3^2 \alpha_2^4 \alpha_3^2 \rho_3 \\
& + 12 x_2 x_3^2 \alpha_2 \rho_4 \alpha_1^5 - 12 x_2 x_3^2 \alpha_2 \rho_1 \alpha_1^5 - 12 x_2 x_3^2 \alpha_2^3 \alpha_3^2 \rho_1 \alpha_1 \\
& + 12 x_2 x_3^2 \alpha_2^3 \alpha_3^2 \rho_4 \alpha_1 - 24 x_2 x_3^2 \alpha_2^3 \alpha_3 \rho_3 \alpha_1^2 + 10 x_2^3 \beta_4 \alpha_2^4 \beta_6 \alpha_3 \\
& - 10 x_2^3 \beta_4 \alpha_2^3 \beta_6 \alpha_1^2 - 3 x_1^3 \beta_2 \beta_3 \alpha_3^4 \alpha_2 - 23 x_1^3 \beta_2 \beta_3 \alpha_3^3 \alpha_1^2 + 10 x_1^3 \beta_4^2 \alpha_1 \alpha_2^3 \alpha_3 \\
& - x_2^3 \beta_2 \beta_4 \alpha_2^4 \alpha_3 + 57 x_1 x_2^2 \beta_4 \alpha_2^3 \beta_5 \alpha_1^2 - 42 x_1 x_2^2 \beta_2 \alpha_2 \beta_6 \alpha_3 \alpha_1^3 \\
& - 2 x_1^3 \beta_2^2 \alpha_3^3 \alpha_1 \alpha_2 + 19 x_2^3 \beta_2 \beta_4 \alpha_2^3 \alpha_1^2 + 9 x_2^3 \beta_2^2 \alpha_3 \alpha_2^2 \alpha_1^2 + 6 x_2^3 \alpha_2^2 \beta_6^2 \alpha_1^2 \alpha_3 \\
& + 48 x_1 x_2^2 \beta_1 \alpha_2^2 \beta_5 \alpha_1^3 - 12 x_1 x_2^2 \beta_1 \alpha_2^3 \beta_5 \alpha_1 \alpha_3 + 18 x_1 x_2^2 \beta_2 \alpha_2^2 \beta_6 \alpha_3^2 \alpha_1 \\
& - 3 x_1 x_2^2 \beta_4 \alpha_2^4 \beta_5 \alpha_3 + 30 x_1 x_2^2 \beta_2^2 \alpha_3 \alpha_2 \alpha_1^3 - 36 x_1 x_2^2 \beta_2 \alpha_2 \beta_5 \alpha_1^4 \\
& + 60 x_1 x_2^2 \beta_2 \beta_4 \alpha_1^3 \alpha_2^2 - 54 x_1 x_2^2 \beta_3 \beta_5 \alpha_1^3 \alpha_2 \alpha_3 + 60 x_1 x_2^2 \beta_1 \alpha_2^2 \beta_6 \alpha_3 \alpha_1^2 \\
& + 6 x_1 x_2^2 \beta_3 \beta_6 \alpha_3^3 \alpha_2^2 + 30 x_1 x_2^2 \beta_3^2 \alpha_3^3 \alpha_1 \alpha_2 - 3 x_1 x_2^2 \beta_1 \alpha_3^2 \beta_2 \alpha_2^3 \\
& + 6 x_1 x_2^2 \beta_1^2 \alpha_2^3 \alpha_1 \alpha_3 - 6 x_1 x_2^2 \beta_1 \beta_4 \alpha_2^4 \alpha_3 - 66 x_1 x_2^2 \beta_1 \beta_4 \alpha_2^3 \alpha_1^2
\end{aligned}$$

$$\begin{aligned}
& -39 x_1 x_2^2 \beta_1 \beta_2 \alpha_1^2 \alpha_2^2 \alpha_3 + 24 x_1 x_2^2 \beta_1 \beta_2 \alpha_1^4 \alpha_2 - 18 x_1 x_2^2 \alpha_2^2 \beta_6^2 \alpha_3^2 \alpha_1 \\
& + 12 x_1 x_2^2 \alpha_2 \beta_6^2 \alpha_3 \alpha_1^3 - 24 x_1 x_2^2 \beta_3 \beta_4 \alpha_1^4 \alpha_2 + 6 x_1 x_2^2 \alpha_2^3 \beta_5^2 \alpha_1 \alpha_3 \\
& + 3 x_1 x_2^2 \alpha_2^3 \beta_6 \alpha_3^2 \beta_5 + 36 x_1 x_2^2 \beta_3 \beta_4 \alpha_2^2 \alpha_3 \alpha_1^2 - 3 x_2^3 \alpha_2^3 \alpha_3 \gamma_2 \alpha_1^2 \\
& - 6 x_2^3 \alpha_2^3 \beta_1^2 \alpha_1^2 - 9 \gamma_1 x_1^3 \alpha_2^3 \alpha_3^3 - 6 x_1^3 \alpha_2 \alpha_3^4 \beta_6 \beta_3 - 16 x_1^3 \alpha_3^2 \beta_3 \alpha_1^3 \beta_1 \\
& + 16 x_1^3 \alpha_3^2 \beta_5 \beta_3 \alpha_1^3 - 3 x_1^3 \alpha_2^2 \alpha_3^3 \beta_6 \beta_5 - 12 x_1 x_3^2 \alpha_2 \rho_3 \alpha_1^5 - 12 x_1 x_3^2 \alpha_2^3 \alpha_3^3 \rho_4 \\
& - 24 x_1 x_3^2 \alpha_2^2 \alpha_3 \rho_3 \alpha_1^3 - 6 x_1 x_3^2 \alpha_2 \mu_4 \mu_3 \alpha_1^4 - 30 x_1 x_2^2 \alpha_2^4 \beta_4^2 \alpha_1 \\
& - 21 x_1 x_2^2 \alpha_2^2 \alpha_3 \gamma_2 \alpha_1^3 - 3 x_1 x_2^2 \alpha_2^2 \alpha_3 \gamma_7 \alpha_1^3 - 27 x_1 x_2^2 \alpha_2^2 \alpha_3^3 \gamma_4 \alpha_1 \\
& + 9 x_1 x_2^2 \alpha_2^4 \alpha_3^2 \gamma_1 + 3 x_1 x_2^2 \alpha_2^4 \alpha_3^2 \gamma_6 + 9 x_1^3 \alpha_3^3 \gamma_4 \alpha_1^3 - 24 x_1 x_2^2 \alpha_2 \gamma_2 \alpha_1^5 \\
& + 12 x_1 x_2^2 \alpha_2 \gamma_7 \alpha_1^5 + 27 x_1 x_2^2 \alpha_2^3 \gamma_5 \alpha_1^3 + 27 x_1 x_2^2 \alpha_2^4 \alpha_3 \gamma_5 \alpha_1 - 9 x_1 x_2^2 \alpha_2^3 \alpha_3^3 \gamma_8 \\
& + 3 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \gamma_2 \alpha_1 + 45 x_1 x_2^2 \alpha_2^3 \alpha_3 \gamma_1 \alpha_1^2 - 21 x_1 x_2^2 \alpha_2^3 \alpha_3 \gamma_6 \alpha_1^2 \\
& - 9 x_2^3 \alpha_2^2 \alpha_3 \gamma_3 \alpha_1^3 + 9 x_2^3 \alpha_2^2 \alpha_3 \gamma_8 \alpha_1^3 - 9 x_2^3 \alpha_2^4 \alpha_3 \gamma_6 \alpha_1 + 3 x_2^3 \alpha_2^4 \alpha_3^2 \gamma_2 \\
& + 3 x_2^3 \alpha_2^3 \alpha_3 \gamma_7 \alpha_1^2 - 9 x_2^3 \alpha_2^3 \alpha_3^2 \gamma_3 \alpha_1 + 20 x_2^3 \alpha_2^4 \beta_4 \beta_5 \alpha_1 - 5 x_2^3 \alpha_2^4 \alpha_3 \beta_1 \beta_5 \\
& + 9 x_2^3 \alpha_2^4 \alpha_3 \gamma_1 \alpha_1 - 20 x_2^3 \alpha_2^4 \beta_1 \beta_4 \alpha_1 + 9 x_2^3 \alpha_2^3 \alpha_3^2 \gamma_8 \alpha_1 + 15 x_2^3 \alpha_2^3 \beta_1 \beta_5 \alpha_1^2 \\
& + 9 x_2^3 \alpha_2^3 \gamma_1 \alpha_1^3 + 9 x_2^3 \alpha_2^4 \gamma_5 \alpha_1^2 - 10 x_2^3 \alpha_2^5 \beta_4^2 + 6 x_1^3 \alpha_2 \alpha_3 \beta_5^2 \alpha_1^3 \\
& + 15 x_1^3 \alpha_2 \alpha_3^3 \gamma_3 \alpha_1^2 - 20 x_1^3 \alpha_2 \alpha_3^2 \beta_6 \alpha_1^2 \beta_1 + 12 x_1^3 \alpha_2 \alpha_3 \beta_1^2 \alpha_1^3 \\
& + 9 x_1^3 \alpha_2 \alpha_3^3 \gamma_8 \alpha_1^2 + 15 x_1^3 \alpha_2 \alpha_3^2 \gamma_2 \alpha_1^3 + 9 x_1^3 \alpha_2 \alpha_3^4 \gamma_4 \alpha_1 + 4 x_1^3 \alpha_2 \alpha_3^3 \beta_3 \beta_1 \alpha_1 \\
& - 4 x_1^3 \alpha_2 \alpha_3^3 \beta_5 \beta_3 \alpha_1 - 4 x_1^3 \alpha_2^2 \alpha_3^2 \beta_5^2 \alpha_1 - 18 x_1^3 \alpha_2 \alpha_3 \beta_5 \beta_1 \alpha_1^3 \\
& - 9 x_1^3 \alpha_2^2 \alpha_3 \gamma_5 \alpha_1^3 + 17 x_1^3 \alpha_2 \alpha_3^2 \beta_5 \beta_6 \alpha_1^2 + 2 x_1^3 \alpha_2^2 \alpha_3^2 \beta_1 \beta_5 \alpha_1 \\
& + 3 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_6 \alpha_1^2 + x_2^2 \alpha_2^2 \alpha_3 \beta_5^2 + 9 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_7 \alpha_1 + 3 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_3 \\
& + 3 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_2 \alpha_1 + 2 x_1^3 \alpha_2^2 \alpha_3^2 \beta_1^2 \alpha_1 + 9 x_1^3 \alpha_2^2 \alpha_3^4 \gamma_8 + 14 x_1^3 \alpha_3^3 \beta_6 \beta_3 \alpha_1^2 \\
& + 6 x_1^3 \alpha_2 \alpha_3 \gamma_6 \alpha_1^4 + 10 x_1^3 \alpha_2 \alpha_3^3 \beta_6^2 \alpha_1 - 21 x_2 x_1^2 \alpha_2 \alpha_3^2 \gamma_3 \alpha_1^3 + 36 x_2 x_1^2 \alpha_2 \gamma_1 \alpha_1^5 \\
& + 9 x_1^3 \alpha_2 \alpha_3^2 \gamma_7 \alpha_1^3 - 9 x_1^3 \alpha_2^3 \alpha_3^2 \gamma_5 \alpha_1 - 12 x_2 x_1^2 \alpha_2 \gamma_6 \alpha_1^5 - 26 x_2 x_1^2 \alpha_2 \alpha_3^2 \beta_6^2 \alpha_1^2 \\
& - 24 x_2 x_1^2 \alpha_2 \alpha_3 \gamma_7 \alpha_1^4 - 9 x_2 x_1^2 \alpha_2 \alpha_3^3 \gamma_4 \alpha_1^2 - 18 x_2 x_1^2 \alpha_2 \alpha_3^2 \beta_3 \beta_1 \alpha_1^2 \\
& + 3 x_2 x_1^2 \alpha_2^3 \alpha_3^3 \gamma_7 + 27 x_2 x_1^2 \alpha_2 \alpha_3^2 \beta_5 \beta_3 \alpha_1^2 + 4 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \beta_6^2 \\
& + 45 x_2 x_1^2 \alpha_2^2 \alpha_3 \gamma_1 \alpha_1^3 + 34 x_2 x_1^2 \alpha_2^2 \beta_5 \beta_4 \alpha_1^3 + 18 x_2 x_1^2 \alpha_2^2 \gamma_5 \alpha_1^4 \\
& - 52 x_2 x_1^2 \alpha_2^2 \beta_1 \beta_4 \alpha_1^3 - 21 x_2 x_1^2 \alpha_2^2 \alpha_3^2 \gamma_7 \alpha_1^2 + 3 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \gamma_3 \alpha_1 \\
& + 9 x_2 x_1^2 \alpha_2^2 \alpha_3^4 \gamma_4 + 10 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \beta_3 \beta_1 - 27 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \gamma_8 \alpha_1 \\
& - 5 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \beta_2 \beta_6 - 32 x_2 x_1^2 \alpha_3 \beta_5 \beta_3 \alpha_1^4 - 24 x_2 x_1^2 \alpha_3 \gamma_3 \alpha_1^5
\end{aligned}$$

$$\begin{aligned}
& -x_2 x_1^2 \alpha_2^2 \alpha_3^3 \beta_5 \beta_3 + x_2 x_1^2 \alpha_2^2 \alpha_3^3 \beta_2^2 + 32 x_2 x_1^2 \alpha_3 \beta_3 \beta_1 \alpha_1^4 - 18 x_2 x_1^2 \alpha_3^2 \gamma_4 \alpha_1^4 \\
& -9 x_2 x_1^2 \alpha_2^4 \alpha_3^2 \gamma_5 + 3 x_2 x_1^2 \alpha_2^2 \alpha_3 \gamma_6 \alpha_1^3 - 15 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \gamma_7 \alpha_1 \\
& + 18 x_1 x_2^2 \alpha_2 \alpha_3 \gamma_8 \alpha_1^4 - 27 x_1 x_2^2 \alpha_2 \alpha_3^2 \gamma_4 \alpha_1^3 - 26 x_2 x_1^2 \alpha_2^3 \alpha_3 \beta_5 \beta_4 \alpha_1 \\
& - 3 x_2^3 \alpha_2^4 \alpha_3^2 \gamma_7 + 36 x_1 x_2^2 \alpha_2^2 \gamma_1 \alpha_1^4 - 24 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_3 \beta_1 \alpha_1 \\
& + 6 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_5 \beta_3 \alpha_1 - 24 x_1 x_2^2 \alpha_2^2 \gamma_6 \alpha_1^4 + 9 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \gamma_8 \alpha_1^2 \\
& + 9 x_2^3 \alpha_2^5 \alpha_3 - 9 x_2^3 \alpha_2^3 \beta_5^2 \alpha_1^2 - 9 x_2^3 \alpha_2^3 \gamma_6 \alpha_1^3 - 6 x_2^3 \alpha_2^2 \gamma_2 \alpha_1^4 \\
& + 6 x_2^3 \alpha_2^2 \gamma_7 \alpha_1^4 + 4 x_2^3 \alpha_2^4 \alpha_3 \beta_1^2 - 3 x_1^3 \alpha_2^3 \alpha_3^3 \gamma_6 + 12 x_1^3 \alpha_3^3 \gamma_3 \alpha_1^4 \\
& - 10 x_1^3 \beta_3^2 \alpha_1 \alpha_3^4 + 12 x_1^3 \alpha_3 \gamma_2 \alpha_1^5 + 8 x_2 x_1^2 \alpha_2^3 \alpha_3 \beta_1 \beta_4 \alpha_1 + 9 x_2 x_1^2 \alpha_2^3 \alpha_3 \gamma_5 \alpha_1^2 \\
& + 9 x_2 x_1^2 \alpha_2^3 \alpha_3^2 \gamma_1 \alpha_1 + 15 x_2 x_1^2 \alpha_2^3 \alpha_3^2 \gamma_6 \alpha_1 - 27 x_2 x_1^2 \alpha_2 \alpha_3^2 \gamma_8 \alpha_1^3 \\
& - 6 \alpha_2^3 x_2 x_3^2 \mu_2 \mu_3 \alpha_3^2 - 3 \alpha_2^2 x_2 x_3^2 \mu_1^2 \alpha_3^2 \alpha_1^2 + 3 \alpha_2^2 x_2 x_3^2 \mu_2^2 \alpha_3^2 \alpha_1^2 \\
& - 3 \alpha_2^3 x_2 x_3^2 \mu_1^2 \alpha_3^2 + 3 \alpha_2^2 x_2 x_3^2 \mu_2^2 \alpha_3^3 - 6 \alpha_2^2 x_2 x_3^2 \alpha_1^3 \mu_3 \mu_4 \\
& - 18 \alpha_2^2 x_2 x_3^2 \mu_2 \mu_3 \alpha_3^2 \alpha_1^2 - 6 \alpha_2^3 x_2 x_3^2 \alpha_1 \mu_3 \mu_4 \alpha_3 - 3 \alpha_2^3 x_2 x_3^2 \mu_4^2 \alpha_3^2 \\
& - 6 \alpha_2^2 x_2 x_3^2 \mu_2 \alpha_3 \alpha_1 \mu_4 - 3 \alpha_2^2 x_2 x_3^2 \mu_4^2 \alpha_3 \alpha_1^2 - 6 \alpha_2^2 x_2 x_3^2 \mu_2 \alpha_3^2 \alpha_1 \mu_4 \\
& + 6 \alpha_2^2 x_2 x_3^2 \mu_1 \mu_2 \alpha_3 \alpha_1^3 + 6 \alpha_2^2 x_2 x_3^2 \mu_1 \mu_2 \alpha_3^2 \alpha_1^2 + 6 \alpha_2^3 x_2 x_3^2 \alpha_1 \mu_3 \mu_1 \alpha_3 \\
& + 6 \alpha_2^2 x_2 x_3^2 \alpha_1^3 \mu_3 \mu_1 + 3 \alpha_2^4 x_2 x_3^2 \mu_3^2 \alpha_3 + 3 \alpha_2^3 x_2 x_3^2 \mu_3^2 \alpha_1^2 \\
& + 6 \alpha_2^3 x_2 x_3^2 \mu_1 \mu_4 \alpha_3^2 - 3 \alpha_1^3 x_1 x_3^2 \alpha_2 \mu_4^2 \alpha_3 + 6 \alpha_2^2 x_2 x_3^2 \mu_1 \mu_4 \alpha_3 \alpha_1^2 \\
& - 3 \alpha_1^3 x_1 x_3^2 \alpha_2^2 \mu_4^2 \alpha_3^2 - 3 \alpha_1^3 x_1 x_3^2 \mu_1^2 \alpha_2^2 \alpha_3^2 - 3 \alpha_1^3 x_1 x_3^2 \mu_1^2 \alpha_2 \alpha_3 \\
& - 24 x_1^2 x_2 \beta_2 \beta_5 \alpha_1^5 + 6 \alpha_1^3 x_1 x_3^2 \mu_1 \alpha_2 \mu_4 \alpha_3 + 6 \alpha_1 x_1 x_3^2 \mu_1 \alpha_2^2 \mu_4 \alpha_3^2 \\
& + 3 \alpha_1^3 x_1 x_3^2 \alpha_2^2 \mu_3^2 + 3 \alpha_1 x_1 x_3^2 \alpha_2^3 \mu_3^2 \alpha_3 - 24 \gamma_2 x_1^2 x_2 \alpha_1^6 \\
& - 18 \alpha_1^3 x_1 x_3^2 \mu_2 \alpha_2 \mu_3 \alpha_3 - 6 \alpha_1 x_1 x_3^2 \mu_2 \alpha_2^2 \mu_3 \alpha_3^2 + 3 \alpha_1 x_1 x_3^2 \mu_2^2 \alpha_3^3 \alpha_2 \\
& + 3 \alpha_1^3 x_1 x_3^2 \mu_2^2 \alpha_3^2 - 12 \alpha_1^5 x_1 x_3^2 \mu_2 \mu_3 + 3 x_1^3 \beta_1 \alpha_3^3 \beta_2 \alpha_2^2) / ((\alpha_2 \alpha_3 + \alpha_1^2) \\
& (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4)), -\frac{1}{24} (-19 x_2^2 \alpha_2 \beta_2 \beta_6 \alpha_3^2 \alpha_1^2 x_1 \\
& - 8 x_1 x_2^2 \alpha_3^3 \beta_6 \beta_3 \alpha_1 \alpha_2 + 3 x_2^3 \alpha_2^3 \alpha_3^2 \beta_6 \beta_5 + 9 x_2^3 \alpha_2^3 \alpha_3 \gamma_1 \alpha_1^2 \\
& + 2 x_2^3 \alpha_2^3 \alpha_3 \beta_5^2 \alpha_1 + 4 x_2^3 \alpha_2^3 \beta_2^2 \alpha_3^2 \alpha_1 + 3 x_2^3 \alpha_2^2 \alpha_3^2 \gamma_3 \alpha_1^2 + 3 x_2^3 \alpha_2^2 \beta_2 \beta_3 \alpha_3^3 \\
& - 16 x_2^3 \alpha_2^2 \beta_2 \beta_4 \alpha_1^3 - 27 x_2^3 \alpha_2^2 \alpha_3^2 \gamma_8 \alpha_1^2 + 9 x_2^3 \alpha_2^2 \alpha_3^2 \gamma_4 \alpha_1 \\
& + 14 x_2^3 \alpha_2^3 \beta_1 \beta_4 \alpha_1^2 - 3 x_2^3 \alpha_2^4 \alpha_3 \beta_5 \beta_4 - 32 x_1 x_2^2 \alpha_3 \beta_4 \beta_3 \alpha_1^3 \alpha_2 \\
& + 20 x_1 x_2^2 \alpha_2^3 \beta_4^2 \alpha_1^2 + x_1 x_2^2 \alpha_2^3 \alpha_3^2 \beta_5^2 - 3 x_1 x_2^2 \alpha_2^3 \alpha_3^3 \gamma_7 + 3 x_1 x_2^2 \alpha_2^3 \alpha_3^3 \gamma_2
\end{aligned}$$



$$\begin{aligned}
& +27 x_2^2 \alpha_2^2 \alpha_3 \beta_2 \beta_4 \alpha_1^2 x_1 - 9 x_1^3 \alpha_2 \alpha_3^2 \gamma_1 \alpha_1^3 + 15 x_2^3 \alpha_2^3 \alpha_3 \gamma_6 \alpha_1^2 \\
& - 20 x_1^3 \alpha_3^4 \beta_2 \beta_3 \alpha_1 - 36 x_1^2 x_2 \alpha_2 \beta_3 \alpha_3^3 \beta_1 \alpha_1 - 10 x_2^3 \alpha_2^3 \alpha_3 \beta_1^2 \alpha_1 \\
& - 18 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_1 \beta_5 \alpha_1 - 3 x_3^2 \alpha_2^2 \mu_3^2 \alpha_1^3 x_2 + 31 x_2^2 \alpha_2^2 \alpha_3 \beta_1 \beta_5 \alpha_1^2 x_1 \\
& - 18 x_2^2 \alpha_2^2 \alpha_3 \beta_6 \alpha_1^2 \beta_4 x_1 - 12 x_3^2 \alpha_2^3 \alpha_3^3 \rho_1 x_2 - 12 x_3^2 \alpha_3 \rho_1 \alpha_1^5 x_1 \\
& + 3 x_1^2 \alpha_2^2 \alpha_3^4 \gamma_3 x_2 + 9 x_1^3 \alpha_2 \alpha_3^4 \gamma_3 \alpha_1 - 18 x_2^2 \alpha_2^2 \gamma_5 \alpha_1^4 x_1 + 12 x_3^2 \rho_3 \alpha_1^5 \alpha_2 x_2 \\
& - 12 x_3^2 \alpha_2^3 \alpha_3^3 \rho_4 x_2 + 24 x_1 x_2^2 \gamma_6 \alpha_1^5 \alpha_2 + 12 x_3^2 \alpha_3 \rho_4 \alpha_1^5 x_1 + 9 x_1^2 \alpha_2^2 \alpha_3^4 \gamma_8 x_2 \\
& - 9 x_1^3 \alpha_2 \alpha_3^4 \gamma_8 \alpha_1 + 10 x_1 x_2^2 \beta_3^2 \alpha_3^4 \alpha_2 + 30 x_1^2 x_2 \beta_3^2 \alpha_3^4 \alpha_1 - 3 x_1^2 \alpha_2^3 \alpha_3^3 \gamma_6 x_2 \\
& + 9 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_6 \alpha_1 - 9 x_1^2 \alpha_2^3 \alpha_3^3 \gamma_1 x_2 - 9 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_1 \alpha_1 - 60 x_3^2 \alpha_3 \rho_4 \alpha_1^4 \alpha_2 x_2 \\
& - 9 x_2^3 \alpha_2^2 \alpha_3 \gamma_2 \alpha_1^3 + 3 x_3^2 \alpha_3 \mu_1^2 \alpha_1^3 \alpha_2 x_2 + 6 x_3^2 \mu_4 \mu_3 \alpha_1^4 \alpha_2 x_2 - 9 x_1^3 \alpha_3^3 \beta_2^2 \alpha_1^2 \\
& + 18 x_1^2 \alpha_3 \gamma_1 \alpha_1^4 \alpha_2 x_2 - 30 x_1^2 \alpha_3 \beta_5^2 \alpha_1^3 \alpha_2 x_2 - 6 x_1^2 \alpha_3^3 \beta_2^2 \alpha_1 \alpha_2 x_2 \\
& + 8 x_2^3 \alpha_2 \beta_3 \beta_4 \alpha_1^4 + 12 x_2^3 \alpha_2 \beta_2 \beta_5 \alpha_1^4 - 6 x_2^3 \alpha_2 \beta_2^2 \alpha_1^3 \alpha_3 - 3 x_2^3 \alpha_2^3 \alpha_3^2 \gamma_7 \alpha_1 \\
& - 10 x_1^3 \alpha_3^3 \beta_3 \beta_1 \alpha_1^2 - 9 x_1 x_2^2 \alpha_2^2 \gamma_4 \alpha_3^4 + 9 x_1^3 \alpha_3^3 \gamma_3 \alpha_1^3 - 10 x_1 x_2^2 \alpha_2^4 \alpha_3 \beta_4^2 \\
& - x_1 x_2^2 \alpha_2^2 \alpha_3^3 \beta_2^2 - 12 x_1^2 x_2 \alpha_3 \gamma_2 \alpha_1^5 - 18 x_2^3 \alpha_3 \gamma_8 \alpha_1^4 \alpha_2 - 36 x_2^2 \alpha_3 \gamma_8 \alpha_1^5 x_1 \\
& - 21 x_1^2 \alpha_3^3 \gamma_3 \alpha_1^2 \alpha_2 x_2 + 9 x_2^3 \alpha_3^2 \gamma_4 \alpha_1^3 \alpha_2 - 30 x_1^2 \alpha_3 \gamma_6 \alpha_1^4 \alpha_2 x_2 \\
& + 3 x_1^2 \alpha_3^2 \gamma_2 \alpha_1^3 \alpha_2 x_2 + 18 x_2^2 \alpha_3^2 \gamma_4 \alpha_1^4 x_1 + 45 x_1^2 \alpha_3^3 \gamma_8 \alpha_1^2 \alpha_2 x_2 \\
& - 10 x_2^3 \alpha_3^3 \beta_3^2 \alpha_1 \alpha_2 - 20 x_2^2 \alpha_3^3 \beta_3^2 \alpha_1^2 x_1 + 6 x_2^3 \alpha_3 \gamma_3 \alpha_1^4 \alpha_2 + 12 x_2^2 \alpha_3 \gamma_3 \alpha_1^5 x_1 \\
& + 6 x_1^3 \alpha_3^2 \gamma_2 \alpha_1^4 - 9 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_5 \alpha_1^2 - 27 x_1^2 x_2 \alpha_3^3 \gamma_4 \alpha_1^3 + 15 x_1^3 \alpha_3^3 \beta_2 \beta_6 \alpha_1^2 \\
& + 9 x_1 x_2^2 \alpha_2^4 \alpha_3^2 \gamma_5 - 3 x_2^3 \alpha_2^3 \alpha_3^2 \beta_1 \beta_2 - 9 x_1^3 \alpha_3^3 \gamma_8 \alpha_1^3 - 23 x_2^3 \alpha_2^3 \beta_5 \beta_4 \alpha_1^2 \\
& + 24 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_6 \beta_4 \alpha_1 - 3 x_2^3 \alpha_2^2 \alpha_3^3 \gamma_3 - 6 x_1^2 \alpha_3^2 \beta_6 \beta_3 \alpha_2 x_2 - 9 x_2^3 \alpha_2^4 \alpha_3 \gamma_5 \alpha_1 \\
& + 16 x_2^3 \alpha_2^2 \beta_6 \beta_4 \alpha_1^3 - 2 x_2^3 \alpha_2^2 \beta_6^2 \alpha_3^2 \alpha_1 + 6 x_2^3 \alpha_2^2 \beta_6 \beta_3 \alpha_3^3 - 6 x_2^3 \alpha_2^4 \alpha_3 \beta_1 \beta_4 \\
& - 6 x_1^3 \alpha_3^2 \beta_5 \beta_6 \alpha_1^3 + 12 x_1^3 \alpha_3^2 \beta_5 \beta_2 \alpha_1^3 - 6 x_1^3 \alpha_3^2 \beta_2 \alpha_1^3 \beta_1 + 9 x_1^3 \alpha_2 \alpha_3^2 \beta_5^2 \alpha_1^2 \\
& + 19 x_1^3 \alpha_3^3 \beta_5 \beta_3 \alpha_1^2 - 4 x_1 x_2^2 \alpha_2^2 \alpha_3^3 \beta_6^2 + 24 x_1^2 x_2 \alpha_3 \gamma_7 \alpha_1^5 + 4 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \beta_1^2 \\
& + 24 x_1^2 x_2 \alpha_3^2 \beta_6^2 \alpha_1^3 + 24 x_1^2 x_2 \alpha_3^2 \beta_2^2 \alpha_1^3 + 3 x_1^3 \alpha_2 \alpha_3^3 \gamma_2 \alpha_1^2 - 3 x_1^3 \alpha_2 \alpha_3^3 \gamma_7 \alpha_1^2 \\
& + 9 x_1^3 \alpha_2 \alpha_3^2 \gamma_6 \alpha_1^3 + 6 x_1^3 \alpha_2 \alpha_3^2 \beta_1^2 \alpha_1^2 + 4 x_2^3 \alpha_2^3 \alpha_3 \beta_2 \beta_4 \alpha_1 \\
& + 8 x_2^3 \alpha_2^3 \alpha_3 \beta_1 \beta_5 \alpha_1 - 4 x_2^3 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \beta_4 - 6 x_1^3 \alpha_3^2 \gamma_7 \alpha_1^4 \\
& + 12 x_3^2 \alpha_2^2 \alpha_3^3 \rho_4 \alpha_1 x_1 - 12 x_1^2 \alpha_3 \beta_1^2 \alpha_1^3 \alpha_2 x_2 + 6 x_3^2 \alpha_2 \alpha_3^3 \mu_1 \mu_2 \alpha_1 x_1 \\
& + 21 x_1^2 \alpha_3^2 \gamma_7 \alpha_1^3 \alpha_2 x_2 - 15 x_1^3 \alpha_2 \alpha_3^2 \beta_5 \beta_1 \alpha_1^2 - 5 x_1^3 \alpha_2 \alpha_3^4 \beta_2 \beta_6 \\
& - 10 x_1^3 \alpha_2^2 \alpha_3^3 \beta_6 \beta_4 + x_1^3 \alpha_2^2 \alpha_3^3 \beta_2 \beta_4 - 9 x_2^2 \alpha_2^2 \alpha_3^3 \gamma_8 \alpha_1 x_1 + 5 x_1^3 \alpha_2^2 \alpha_3^3 \beta_1 \beta_5
\end{aligned}$$

$$\begin{aligned}
& -x_1^3 \alpha_2 \beta_5 \beta_3 \alpha_3^4 - 9x_2^3 \alpha_2^3 \alpha_3^2 \gamma_2 \alpha_1 - 24x_3^2 \alpha_2^2 \alpha_3^2 \rho_1 \alpha_1^2 x_2 \\
& - 24x_3^2 \alpha_2 \alpha_3^2 \rho_1 \alpha_1^3 x_1 - 3x_3^2 \alpha_3^2 \mu_2^2 \alpha_1^3 x_2 + 3x_3^2 \alpha_3^3 \mu_2^2 \alpha_1^2 x_1 \\
& + 12x_3^2 \alpha_3 \rho_2 \alpha_1^5 x_2 - 12x_3^2 \alpha_3^2 \rho_2 \alpha_1^4 x_1 - 12x_3^2 \alpha_2^3 \alpha_3^3 \rho_3 x_1 - 12x_3^2 \alpha_2^2 \alpha_3^4 \rho_2 x_1 \\
& + 3x_3^2 \alpha_2 \alpha_3^4 \mu_2^2 x_1 + 36x_1^2 x_2 \alpha_3^2 \gamma_8 \alpha_1^4 - 9x_2^3 \alpha_2^3 \alpha_3^3 \gamma_8 - 20x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_2 \alpha_1 \beta_1 \\
& + 10x_1^3 \alpha_2^3 \alpha_3^2 \beta_4^2 + 9x_1^3 \alpha_2 \gamma_4 \alpha_3^5 - 45x_1^2 x_2 \alpha_2 \alpha_3^2 \beta_2 \alpha_1^2 \beta_1 \\
& + 6x_3^2 \alpha_2^2 \alpha_3^2 \mu_2 \mu_3 \alpha_1 x_2 - 18x_3^2 \alpha_2 \alpha_3^2 \mu_2 \mu_3 \alpha_1^2 x_1 - 3x_3^2 \alpha_2^3 \alpha_3^2 \mu_3^2 \alpha_1 x_2 \\
& + 3x_3^2 \alpha_2^2 \alpha_3 \mu_3^2 \alpha_1^2 x_1 + 10x_2^3 \alpha_2^4 \beta_4^2 \alpha_1 + 3x_2^3 \alpha_2^4 \alpha_3^2 \gamma_6 + 12x_2^3 \alpha_2^2 \gamma_6 \alpha_1^4 \\
& + 9x_2^3 \alpha_2^4 \alpha_3^2 \gamma_1 - 6x_1^3 \alpha_3^3 \beta_6^2 \alpha_1^2 + 6x_3^2 \alpha_2 \alpha_3 \alpha_1^3 \mu_1 \mu_3 x_1 - 6x_3^2 \alpha_2 \alpha_3 \alpha_1^3 \mu_4 \mu_3 x_1 \\
& - 9x_2^3 \alpha_2^3 \gamma_5 \alpha_1^3 - 6x_3^2 \alpha_2^2 \alpha_3^2 \mu_4 \mu_1 \alpha_1 x_2 + 6x_3^2 \alpha_2 \alpha_3^2 \mu_4 \mu_1 \alpha_1^2 x_1 \\
& - 4x_1^3 \alpha_2^2 \alpha_3^3 \beta_1^2 + 4x_1^3 \alpha_2 \alpha_3^4 \beta_6^2 - x_1^3 \alpha_2^2 \alpha_3^3 \beta_5^2 - 3x_1^3 \alpha_2^2 \alpha_3^4 \gamma_2 \\
& + 3x_1^3 \alpha_2^2 \alpha_3^4 \gamma_7 + 9x_1^3 \alpha_3^4 \gamma_4 \alpha_1^2 - 48x_3^2 \alpha_2^2 \alpha_3^2 \rho_4 \alpha_1^2 x_2 + 24x_3^2 \alpha_2^2 \alpha_3^2 \rho_4 \alpha_1^3 x_1 \\
& + 12x_2^3 \alpha_2^2 \beta_5^2 \alpha_1^3 - 12x_2^3 \alpha_2 \gamma_7 \alpha_1^5 + 12x_2 x_3^2 \mu_2 \mu_3 \alpha_1^5 - 8x_1^3 \beta_5 \alpha_3^3 \beta_2 \alpha_1 \alpha_2 \\
& - 6x_1^2 \alpha_2^2 \alpha_3^2 \beta_4 \beta_2 \alpha_1 x_2 - 12x_3^2 \alpha_2^2 \alpha_3^2 \rho_1 \alpha_1 x_1 + 6x_2 x_3^2 \mu_2 \alpha_2 \mu_4 \alpha_3^2 \alpha_1^2 \\
& - 6\alpha_3^2 x_1 x_3^2 \alpha_2^2 \mu_4 \alpha_1 \mu_3 + 6\alpha_3^2 x_1 x_3^2 \mu_1 \alpha_2^2 \mu_3 \alpha_1 - 3\alpha_3^3 x_1 x_3^2 \alpha_2^2 \mu_4^2 \\
& - 3\alpha_3^3 x_1 x_3^2 \alpha_2^2 \mu_1^2 + 16x_2^3 \alpha_3^2 \beta_1 \alpha_1 \alpha_2^2 \beta_3 - 12x_3^2 \alpha_3 \rho_1 \alpha_1^4 \alpha_2 x_2 \\
& + 12x_1^2 x_2 \alpha_2 \alpha_3^3 \beta_2 \beta_6 \alpha_1 + 5x_1 x_2^2 \alpha_2^2 \alpha_3^3 \beta_2 \beta_6 + 6\alpha_3^3 x_1 x_3^2 \alpha_2^2 \mu_4 \mu_1 \\
& - 6\alpha_3^3 x_1 x_3^2 \mu_2 \alpha_2^2 \mu_3 - 6x_2 x_3^2 \mu_1 \mu_2 \alpha_3^2 \alpha_2 \alpha_1^2 - 6x_2 x_3^2 \mu_1 \alpha_2^2 \mu_3 \alpha_3 \alpha_1^2 \\
& + 6x_2 x_3^2 \alpha_2^2 \mu_4 \alpha_3 \mu_3 \alpha_1^2 - 12x_1 x_2^2 \beta_2^2 \alpha_1^4 \alpha_3 + 16x_1 x_2^2 \beta_3 \beta_4 \alpha_1^5 \\
& - 24x_1 x_2^2 \beta_6^2 \alpha_1^4 \alpha_3 + 24x_1 x_2^2 \beta_2 \beta_5 \alpha_1^5 + 21x_2^2 \alpha_2^2 \alpha_3 \gamma_6 \alpha_1^3 x_1 \\
& + 27x_2^2 \alpha_2^3 \alpha_3^2 \gamma_1 \alpha_1 x_1 - 6x_1^2 \alpha_3^3 \beta_6^2 \alpha_1 \alpha_2 x_2 + 17x_2^3 \alpha_2^2 \beta_1 \beta_2 \alpha_1^2 \alpha_3 \\
& - 3x_1^2 \alpha_3^4 \beta_2 \beta_3 \alpha_2 x_2 + 11x_2^3 \alpha_2^2 \beta_5 \beta_6 \alpha_1^2 \alpha_3 + 3x_3^2 \alpha_2^2 \alpha_3^2 \mu_4^2 \alpha_1 x_2 \\
& - 3x_3^2 \alpha_2 \alpha_3^2 \mu_4^2 \alpha_1^2 x_1 - 20x_2^3 \alpha_2^2 \beta_6 \alpha_3 \beta_1 \alpha_1^2 + 2x_2^3 \alpha_2^2 \beta_5 \beta_3 \alpha_3^2 \alpha_1 \\
& - 6x_3^2 \mu_3 \alpha_1^4 \mu_1 \alpha_2 x_2 - 8x_2^3 \alpha_2^2 \beta_2 \beta_5 \alpha_3 \alpha_1^2 - 12x_3^2 \alpha_3 \alpha_1^4 \mu_2 \mu_3 x_1 \\
& + 12x_3^2 \alpha_2^3 \alpha_3^2 \rho_3 \alpha_1 x_2 - 24x_3^2 \alpha_2^2 \alpha_3^2 \rho_3 \alpha_1^2 x_1 + 6x_3^2 \alpha_3^2 \mu_1 \mu_2 \alpha_1^3 x_1 \\
& - 6x_3^2 \alpha_3^2 \mu_2 \mu_4 \alpha_1^3 x_1 + 24x_1 x_2^2 \alpha_2 \beta_5^2 \alpha_1^4 - 24x_1 x_2^2 \beta_5 \beta_6 \alpha_1^5 \\
& - 24x_1 x_2^2 \beta_1 \alpha_2 \beta_5 \alpha_1^4 + 36x_1 x_2^2 \beta_2 \beta_6 \alpha_1^4 \alpha_3 + 9x_1^2 \alpha_2^2 \alpha_3^2 \gamma_1 \alpha_1^2 x_2 \\
& - 15x_2^3 \alpha_2^2 \alpha_3 \gamma_7 \alpha_1^3 - 3x_1^2 x_2 \alpha_2^2 \alpha_3^3 \gamma_7 \alpha_1 + 15x_1^2 x_2 \alpha_2^2 \alpha_3^3 \gamma_2 \alpha_1 \\
& + 8x_1^3 \alpha_3^2 \beta_4 \beta_3 \alpha_1^3 + 3x_3^2 \alpha_3 \mu_4^2 \alpha_1^3 \alpha_2 x_2 - 12x_3^2 \alpha_2 \alpha_3 \rho_3 \alpha_1^4 x_1
\end{aligned}$$

$$\begin{aligned}
& -45 x_2^2 \alpha_2 \alpha_3^2 \gamma_8 \alpha_1^3 x_1 - 3 x_2^2 \alpha_2 \alpha_3^2 \gamma_3 \alpha_1^3 x_1 + 9 x_2^2 \alpha_2 \alpha_3^3 \gamma_4 \alpha_1^2 x_1 \\
& + 28 x_2^2 \alpha_2^2 \beta_1 \beta_4 \alpha_1^3 x_1 + 3 x_3^2 \alpha_2^2 \alpha_3^2 \mu_1^2 \alpha_1 x_2 - 3 x_3^2 \alpha_2 \alpha_3^2 \mu_1^2 \alpha_1^2 x_1 \\
& + 24 x_3^2 \alpha_2^2 \alpha_3 \rho_3 \alpha_1^3 x_2 - 33 x_1^2 \alpha_2^2 \alpha_3^2 \gamma_6 \alpha_1^2 x_2 - 12 x_1^3 \alpha_2 \alpha_3^3 \beta_3 \beta_4 \alpha_1 \\
& - 6 x_3^2 \alpha_3 \mu_4 \mu_1 \alpha_1^3 \alpha_2 x_2 + 27 x_2^2 \alpha_2^2 \alpha_3 \gamma_1 \alpha_1^3 x_1 - 5 x_2^2 \alpha_2^2 \alpha_3 \beta_5^2 \alpha_1^2 x_1 \\
& + 22 x_2^3 \alpha_3 \beta_5 \beta_3 \alpha_1^3 \alpha_2 + 44 x_2^2 \alpha_3 \beta_5 \beta_3 \alpha_1^4 x_1 + 26 x_2^3 \alpha_3^2 \beta_6 \beta_3 \alpha_1^2 \alpha_2 \\
& + 52 x_2^2 \alpha_3^2 \beta_6 \beta_3 \alpha_1^3 x_1 - 17 x_2^3 \alpha_3^2 \beta_2 \beta_3 \alpha_1^2 \alpha_2 - 34 x_2^2 \alpha_3^2 \beta_2 \beta_3 \alpha_1^3 x_1 \\
& + 17 x_2^2 \alpha_2 \beta_2^2 \alpha_3^2 \alpha_1^2 x_1 - 4 x_2^3 \alpha_3 \beta_3 \alpha_1^3 \beta_1 \alpha_2 - 8 x_2^2 \alpha_3 \beta_3 \alpha_1^4 \beta_1 x_1 \\
& - 32 x_2^2 \alpha_2 \beta_2 \beta_4 \alpha_1^4 x_1 + 27 x_1^2 \alpha_2^2 \alpha_3 \gamma_5 \alpha_1^3 x_2 + 3 x_1^2 \alpha_2^3 \alpha_3^2 \beta_5 \beta_4 x_2 \\
& - 20 x_1^3 \alpha_2^2 \alpha_3^2 \beta_5 \beta_4 \alpha_1 + 3 x_1^2 \alpha_2^2 \alpha_3^3 \beta_1 \beta_2 x_2 + 14 x_1^3 \alpha_2 \alpha_3^3 \beta_1 \beta_2 \alpha_1 \\
& + 18 x_1^2 \alpha_2^2 \alpha_3^2 \beta_1^2 \alpha_1 x_2 - 3 x_1^2 \alpha_2^2 \alpha_3^3 \beta_6 \beta_5 x_2 + 14 x_1^3 \alpha_2 \alpha_3^3 \beta_6 \beta_5 \alpha_1 \\
& - 20 x_1^3 \alpha_2 \alpha_3^3 \beta_6 \beta_1 \alpha_1 - 6 x_3^2 \alpha_2 \alpha_3^3 \mu_2 \mu_4 \alpha_1 x_1 - 3 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \gamma_6 \alpha_1 \\
& - 5 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \beta_1 \beta_5 - 24 x_1 x_2^2 \alpha_3 \gamma_7 \alpha_1^4 \alpha_2 + 2 x_1 x_2^2 \alpha_3^2 \beta_6^2 \alpha_1^2 \alpha_2 \\
& + 32 x_1 x_2^2 \beta_6 \alpha_1^4 \beta_4 \alpha_2 + 18 x_3^2 \alpha_3 \alpha_1^3 \mu_2 \mu_3 \alpha_2 x_2 - 9 x_1 x_2^2 \alpha_2^3 \alpha_3 \gamma_5 \alpha_1^2 \\
& + 10 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \beta_6 \beta_4 + 24 x_1^2 x_2 \alpha_3 \beta_5 \beta_6 \alpha_1^4 - 48 x_1^2 x_2 \alpha_3^2 \beta_2 \beta_6 \alpha_1^3 \\
& - 66 x_1^2 x_2 \alpha_3^3 \beta_6 \beta_3 \alpha_1^2 - 2 x_2^3 \alpha_2^2 \beta_2 \beta_6 \alpha_3^2 \alpha_1 + 12 x_1^2 x_2 \alpha_3 \beta_2 \alpha_1^4 \beta_1 \\
& - 36 x_1^2 x_2 \alpha_3 \beta_5 \beta_2 \alpha_1^4 + 24 x_1^2 x_2 \alpha_3^2 \beta_3 \beta_1 \alpha_1^3 - 24 x_1^2 x_2 \alpha_3 \beta_4 \beta_3 \alpha_1^4 \\
& - 10 x_1 x_2^2 \alpha_2^2 \beta_3 \alpha_3^3 \beta_1 - 26 x_1 x_2^2 \alpha_2^2 \alpha_3 \beta_1^2 \alpha_1^2 + 27 x_1^2 x_2 \alpha_2^3 \alpha_3^2 \gamma_5 \alpha_1 \\
& + x_1 x_2^2 \alpha_2^2 \beta_5 \beta_3 \alpha_3^3 - x_1 x_2^2 \alpha_2^3 \alpha_3^2 \beta_2 \beta_4 - 19 x_1^3 \alpha_2 \alpha_3^2 \beta_4 \beta_2 \alpha_1^2 \\
& + 10 x_1^3 \alpha_2 \alpha_3^2 \beta_4 \beta_6 \alpha_1^2 - 60 x_1^2 x_2 \alpha_3^2 \beta_5 \beta_3 \alpha_1^3 - 3 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \gamma_7 \alpha_1^2 \\
& - 21 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \gamma_2 \alpha_1^2 + 20 x_1^3 \alpha_3^4 \beta_6 \beta_3 \alpha_1 + 57 x_1^2 x_2 \alpha_3^3 \beta_2 \beta_3 \alpha_1^2 \\
& - 46 x_2^2 \alpha_2^2 \beta_5 \beta_4 \alpha_1^3 x_1 - 27 x_1^2 x_2 \alpha_2 \gamma_4 \alpha_3^4 \alpha_1 - 28 x_2^2 \alpha_2 \beta_2 \beta_5 \alpha_3 \alpha_1^3 x_1 \\
& - 24 x_1 x_2^2 \alpha_3 \gamma_2 \alpha_1^4 \alpha_2 - 15 x_2^2 \alpha_2^2 \alpha_3^3 \gamma_3 \alpha_1 x_1 - 30 x_1^2 \alpha_2^3 \alpha_3 \beta_4^2 \alpha_1 x_2 \\
& + 6 x_1^2 \alpha_2^3 \alpha_3^2 \beta_1 \beta_4 x_2 + 20 x_1^3 \alpha_2^2 \alpha_3^2 \beta_1 \beta_4 \alpha_1 - 32 x_1 x_2^2 \alpha_2^3 \alpha_3 \beta_1 \beta_4 \alpha_1 \\
& - 54 x_1^2 x_2 \alpha_2^2 \alpha_3 \beta_1 \beta_4 \alpha_1^2 + 14 x_1 x_2^2 \alpha_2^3 \alpha_3 \beta_5 \beta_4 \alpha_1 + 63 x_1^2 x_2 \alpha_2^2 \alpha_3 \beta_5 \beta_4 \alpha_1^2 \\
& + 40 x_2^2 \alpha_2 \beta_1 \beta_2 \alpha_1^3 \alpha_3 x_1 + 42 x_2^2 \alpha_2 \beta_3 \alpha_3^2 \alpha_1^2 \beta_1 x_1 + 28 x_2^2 \alpha_2 \beta_5 \beta_6 \alpha_1^3 \alpha_3 x_1 \\
& - 40 x_2^2 \alpha_2 \beta_6 \alpha_3 \beta_1 \alpha_1^3 x_1 - 15 x_2^2 \alpha_2 \beta_5 \beta_3 \alpha_3^2 \alpha_1^2 x_1 - 39 x_1^2 x_2 \alpha_2 \beta_5 \beta_6 \alpha_3^2 \alpha_1^2 \\
& + 6 x_3^2 \alpha_3 \mu_2 \mu_4 \alpha_1^4 x_2 - 12 x_2^3 \alpha_2^2 \beta_1 \beta_5 \alpha_1^3 + 42 x_1^2 x_2 \alpha_2 \beta_5 \alpha_3 \beta_1 \alpha_1^3 \\
& - 36 x_1^2 x_2 \alpha_2 \beta_6 \alpha_1^3 \alpha_3 \beta_4 - 12 x_2^3 \alpha_2 \beta_6^2 \alpha_1^3 \alpha_3 + 18 x_2^3 \alpha_2 \beta_2 \beta_6 \alpha_1^3 \alpha_3
\end{aligned}$$

$$\begin{aligned}
& + 54 x_1^2 x_2 \alpha_2 \beta_4 \beta_2 \alpha_1^3 \alpha_3 + 12 x_1 x_2^2 \alpha_2^2 \beta_4 \beta_3 \alpha_3^2 \alpha_1 + 36 x_1^2 x_2 \alpha_2 \beta_4 \beta_3 \alpha_3^2 \alpha_1^2 \\
& + 8 x_1 x_2^2 \alpha_2^2 \beta_5 \alpha_3^2 \beta_2 \alpha_1 + 24 x_1^2 x_2 \alpha_2 \beta_5 \alpha_3^2 \beta_2 \alpha_1^2 - 8 x_1 x_2^2 \alpha_2^2 \beta_5 \beta_6 \alpha_3^2 \alpha_1 \\
& - 24 x_2 x_1^2 \alpha_3^2 \gamma_3 \alpha_1^4 - 9 x_1^3 \alpha_2^3 \alpha_3^3 \gamma_5 + 20 x_1 x_2^2 \alpha_2^2 \beta_6 \alpha_3^2 \alpha_1 \beta_1 \\
& - 12 x_2^3 \alpha_2 \beta_5 \beta_6 \alpha_1^4 + x_1^3 \alpha_2 \alpha_3^4 \beta_2^2 + 60 x_1^2 x_2 \alpha_2 \beta_6 \alpha_3^2 \alpha_1^2 \beta_1 - 10 x_1^3 \beta_3^2 \alpha_3^5 \\
& + 3 x_3^2 \alpha_2^3 \alpha_3^2 \mu_3^2 x_1 + 10 x_1^3 \alpha_2 \beta_3 \alpha_3^4 \beta_1 - 24 \rho_4 x_2 x_3^2 \alpha_1^6 - 24 \gamma_7 x_1 x_2^2 \alpha_1^6 \\
& + 12 x_3^2 \alpha_2^2 \alpha_3^3 \rho_2 \alpha_1 x_2 - 3 x_3^2 \alpha_2 \alpha_3^3 \mu_2^2 \alpha_1 x_2 + 26 x_2^2 \alpha_2 \beta_2 \beta_3 \alpha_3^3 \alpha_1 x_1 \\
& + 24 x_3^2 \alpha_2 \alpha_3^2 \rho_2 \alpha_1^3 x_2 - 24 x_3^2 \alpha_2 \alpha_3^3 \rho_2 \alpha_1^2 x_1 - 6 x_3^2 \alpha_3 \mu_1 \mu_2 \alpha_1^4 x_2 \\
& - 12 x_2^3 \alpha_2^2 \alpha_3 \beta_3 \beta_4 \alpha_1^2) / ((\alpha_2 \alpha_3 + \alpha_1^2)(\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4)), 0]
\end{aligned}$$

### Output of intermediate results

```

> for j from 1 to targetspace-1 do
  print("Poincare-Dulac normal form up to degree", j, ":");
  for i from 1 to targetspace do
    print(dgl[j,i]);
  od;
od;

```

"Poincare-Dulac normal form up to degree", 1, ":"

$$[\alpha_1 x_1 + \alpha_2 x_2, \alpha_3 x_1 - \alpha_1 x_2, 0]$$

$$[\beta_1 x_1^2 + \beta_2 x_1 x_2 + \beta_3 x_2^2 + \mu_1 x_3 x_1 + \mu_2 x_2 x_3,$$

$$\beta_4 x_1^2 + \beta_5 x_1 x_2 + \beta_6 x_2^2 + \mu_3 x_3 x_1 + \mu_4 x_2 x_3, 0]$$

$$\begin{aligned}
& [\gamma_1 x_1^3 + \gamma_2 x_1^2 x_2 + \gamma_3 x_1 x_2^2 + \gamma_4 x_2^3 + \nu_1 x_1^2 x_3 + \nu_2 x_1 x_2 x_3 + \nu_3 x_2^2 x_3 + \rho_1 x_1 x_3^2 \\
& + \rho_2 x_2 x_3^2, \gamma_5 x_1^3 + \gamma_6 x_1^2 x_2 + \gamma_7 x_1 x_2^2 + \gamma_8 x_2^3 + \nu_4 x_1^2 x_3 + \nu_5 x_1 x_2 x_3 + \nu_6 x_2^2 x_3 \\
& + \rho_3 x_1 x_3^2 + \rho_4 x_2 x_3^2, 0]
\end{aligned}$$

"Poincare-Dulac normal form up to degree", 2, ":"

$$[\alpha_1 x_1 + \alpha_2 x_2, \alpha_3 x_1 - \alpha_1 x_2, 0]$$

$$\left[ \frac{1}{2} x_3 (\alpha_2 \alpha_3 \mu_1 x_1 + 2 \mu_1 x_1 \alpha_1^2 + \alpha_2 \alpha_3 \mu_2 x_2 + \alpha_2 x_2 \mu_1 \alpha_1 - \alpha_2 x_2 \mu_4 \alpha_1 + \mu_3 x_2 \alpha_2^2$$

$$+ \alpha_2 x_1 \mu_3 \alpha_1 + \alpha_3 x_1 \mu_2 \alpha_1 + \alpha_2 \alpha_3 x_1 \mu_4) / (\alpha_2 \alpha_3 + \alpha_1^2), \frac{1}{2} x_3 (\alpha_2 \alpha_3 \mu_3 x_1$$

$$+ \alpha_2 \alpha_3 \mu_4 x_2 + 2 \mu_4 x_2 \alpha_1^2 + \alpha_2 \alpha_3 x_2 \mu_1 - \alpha_2 \mu_3 \alpha_1 x_2 - \alpha_3 \mu_2 \alpha_1 x_2 + \alpha_3 \mu_1 \alpha_1 x_1$$

$$+ x_1 \mu_2 \alpha_3^2 - \alpha_3 \mu_4 \alpha_1 x_1) / (\alpha_2 \alpha_3 + \alpha_1^2), 0]$$

$$\left[ \frac{1}{12} (-18 \beta_2 \mu_2 \alpha_3 \alpha_1 \alpha_2 + 5 \beta_2 \alpha_2 \mu_4 \alpha_1^2 + 6 \beta_2 \mu_2 \alpha_1^3 - 4 \mu_1 \beta_3 \alpha_1^3 - 21 \beta_3 \mu_2 \alpha_3^2 \alpha_2$$

$$+ 9 \beta_3 \mu_3 \alpha_1^2 \alpha_2 + 9 \beta_3 \alpha_2^2 \mu_3 \alpha_3 + 24 \mu_2 \beta_5 \alpha_1^2 \alpha_2 + 18 \mu_2 \alpha_2 \beta_6 \alpha_3 \alpha_1 - 24 \mu_2 \beta_4 \alpha_1 \alpha_2^2$$

$$\begin{aligned}
& -24 \beta_1 \alpha_1^2 \alpha_2 \mu_2 - 11 \beta_2 \alpha_2^2 \mu_4 \alpha_3 + 7 \beta_2 \mu_1 \alpha_2^2 \alpha_3 - \beta_2 \mu_1 \alpha_1^2 \alpha_2 + 8 \beta_3 \alpha_1^3 \mu_4 \\
& + 8 \mu_1 \beta_4 \alpha_2^3 + 8 \mu_1 \beta_1 \alpha_2^2 \alpha_1 - 12 \mu_1 \beta_3 \alpha_3 \alpha_1 \alpha_2 - 8 \mu_1 \alpha_2^2 \beta_5 \alpha_1 - 7 \mu_1 \alpha_2^2 \beta_6 \alpha_3 \\
& + \mu_1 \alpha_2 \beta_6 \alpha_1^2 + 3 \beta_3 \mu_2 \alpha_3 \alpha_1^2 - 6 \mu_2 \beta_6 \alpha_1^3 - 5 \alpha_2 \mu_4 \alpha_1^2 \beta_6 - 16 \beta_1 \alpha_2^2 \alpha_1 \mu_4 \\
& + 24 \beta_3 \alpha_3 \alpha_1 \alpha_2 \mu_4 - 16 \beta_4 \alpha_2^3 \mu_4 + 11 \alpha_2^2 \mu_4 \alpha_3 \beta_6 + 16 \alpha_2^2 \beta_5 \alpha_1 \mu_4) x_2^2 x_3 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \rho_2 x_2 x_3^2 + \rho_1 x_1 x_3^2 + v_3 x_2^2 x_3 + \frac{1}{12} (16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \beta_4 \\
& - 4 \beta_2 \beta_5 \alpha_3 \alpha_1^2 - 4 \beta_1 \alpha_3^2 \beta_2 \alpha_2 - 4 \beta_5 \alpha_3 \alpha_2^2 \beta_4 - 8 \beta_1 \beta_4 \alpha_2^2 \alpha_3 + 4 \beta_1 \beta_2 \alpha_1^2 \alpha_3 \\
& + 8 \beta_2 \beta_4 \alpha_2 \alpha_3 \alpha_1 - 16 \beta_4^2 \alpha_1 \alpha_2^2 + 4 \beta_2 \beta_5 \alpha_3^2 \alpha_2 + 12 \alpha_2 \beta_5 \alpha_1^2 \beta_4 - 8 \beta_2 \beta_6 \alpha_3^2 \alpha_1 \\
& + 16 \beta_2 \beta_4 \alpha_1^3 + 8 \beta_2 \beta_3 \alpha_3^3 + 8 \beta_2^2 \alpha_3^2 \alpha_1 - 8 \beta_3 \alpha_3^2 \alpha_2 \beta_4 + 8 \beta_3 \alpha_3 \alpha_1^2 \beta_4 \\
& - 24 \beta_1 \beta_4 \alpha_1^2 \alpha_2) x_1^3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + v_1 x_1^2 x_3 + \gamma_3 x_1 x_2^2 \\
& + v_2 x_1 x_2 x_3 + \frac{1}{24} (6 \mu_1 \alpha_2^2 \mu_4 \alpha_3 + 6 \alpha_2^2 \alpha_1 \mu_3 \mu_1 - 6 \mu_2 \alpha_2^2 \mu_3 \alpha_3 - 6 \alpha_2^2 \alpha_1 \mu_3 \mu_4 \\
& + 3 \mu_2^2 \alpha_3^2 \alpha_2 - 12 \mu_2 \mu_3 \alpha_1^2 \alpha_2 + 6 \mu_1 \mu_2 \alpha_3 \alpha_1 \alpha_2 - 6 \mu_2 \alpha_3 \alpha_1 \alpha_2 \mu_4 + 3 \alpha_2^3 \mu_3^2 \\
& - 3 \mu_1^2 \alpha_2^2 \alpha_3 - 3 \alpha_2^2 \mu_4^2 \alpha_3) x_2 x_3^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} ( \\
& 6 \mu_1 \alpha_2 \mu_3 \alpha_1^2 + 6 \alpha_2 \alpha_3 \alpha_1 \mu_4 \mu_1 + 6 \mu_1 \mu_2 \alpha_3 \alpha_1^2 + 3 \alpha_2^2 \alpha_1 \mu_3^2 - 3 \alpha_2 \alpha_3 \alpha_1 \mu_1^2 \\
& - 12 \mu_2 \mu_3 \alpha_1^3 - 6 \mu_2 \mu_4 \alpha_3 \alpha_1^2 + 3 \alpha_3^2 \alpha_1 \mu_2^2 - 6 \alpha_2 \mu_4 \alpha_1^2 \mu_3 - 3 \alpha_2 \alpha_3 \alpha_1 \mu_4^2 \\
& - 6 \mu_2 \alpha_2 \mu_3 \alpha_3 \alpha_1) x_1 x_3^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} (18 \beta_2 \mu_2 \alpha_3 \alpha_1 \alpha_2 \\
& - 3 \beta_2 \alpha_2 \mu_4 \alpha_1^2 - 6 \beta_2 \mu_2 \alpha_1^3 + 15 \beta_3 \mu_2 \alpha_3^2 \alpha_2 - 15 \beta_3 \mu_3 \alpha_1^2 \alpha_2 - 15 \beta_3 \alpha_2^2 \mu_3 \alpha_3 \\
& - 24 \mu_2 \beta_5 \alpha_1^2 \alpha_2 - 18 \mu_2 \alpha_2 \beta_6 \alpha_3 \alpha_1 + 24 \mu_2 \beta_4 \alpha_1 \alpha_2^2 + 24 \beta_1 \alpha_1^2 \alpha_2 \mu_2 \\
& + 9 \beta_2 \alpha_2^2 \mu_4 \alpha_3 - 9 \beta_2 \mu_1 \alpha_2^2 \alpha_3 + 3 \beta_2 \mu_1 \alpha_1^2 \alpha_2 - 12 \mu_1 \beta_4 \alpha_2^3 - 12 \mu_1 \beta_1 \alpha_2^2 \alpha_1 \\
& + 12 \mu_1 \beta_3 \alpha_3 \alpha_1 \alpha_2 + 12 \mu_1 \alpha_2^2 \beta_5 \alpha_1 + 9 \mu_1 \alpha_2^2 \beta_6 \alpha_3 - 3 \mu_1 \alpha_2 \beta_6 \alpha_1^2 - 9 \beta_3 \mu_2 \alpha_3 \alpha_1^2 \\
& + 6 \mu_2 \beta_6 \alpha_1^3 + 3 \alpha_2 \mu_4 \alpha_1^2 \beta_6 + 12 \beta_1 \alpha_2^2 \alpha_1 \mu_4 - 12 \beta_3 \alpha_3 \alpha_1 \alpha_2 \mu_4 + 12 \beta_4 \alpha_2^3 \mu_4 \\
& - 9 \alpha_2^2 \mu_4 \alpha_3 \beta_6 - 12 \alpha_2^2 \beta_5 \alpha_1 \mu_4) x_2^2 x_3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} ( \\
& -8 \alpha_2^2 \mu_1 \beta_4 \alpha_1 + 8 \alpha_2 \alpha_3 \mu_1 \beta_6 \alpha_1 - 8 \alpha_2 \alpha_3 \mu_1 \beta_2 \alpha_1 - 10 \beta_1 \mu_1 \alpha_2^2 \alpha_3 + 16 \mu_2 \beta_4 \alpha_1^2 \alpha_2 \\
& + 10 \mu_1 \beta_3 \alpha_3 \alpha_1^2 + \mu_1 \beta_5 \alpha_3 \alpha_2^2 - \beta_2 \alpha_2^2 \mu_3 \alpha_3 + \beta_2 \mu_2 \alpha_3^2 \alpha_2 + 33 \beta_2 \mu_2 \alpha_3 \alpha_1^2 \\
& - 17 \beta_2 \alpha_2 \mu_3 \alpha_1^2 - 10 \beta_3 \mu_4 \alpha_3 \alpha_1^2 + 32 \mu_2 \beta_3 \alpha_3^2 \alpha_1 + 8 \mu_2 \beta_6 \alpha_3^2 \alpha_2 \\
& - 36 \beta_3 \alpha_2 \mu_3 \alpha_3 \alpha_1 - 18 \mu_2 \beta_5 \alpha_1^3 - 24 \mu_2 \beta_6 \alpha_1^2 \alpha_3 - 16 \mu_2 \beta_4 \alpha_2^2 \alpha_3 \\
& + 14 \mu_2 \beta_5 \alpha_3 \alpha_1 \alpha_2 - 18 \beta_1 \mu_1 \alpha_1^2 \alpha_2 + 10 \beta_1 \alpha_2^2 \mu_4 \alpha_3 + 4 \beta_1 \mu_2 \alpha_3 \alpha_1 \alpha_2
\end{aligned}$$

$$\begin{aligned}
& + 18 \beta_1 \alpha_2 \mu_4 \alpha_1^2 + 36 \beta_1 \mu_2 \alpha_1^3 - \alpha_2^2 \mu_4 \alpha_3 \beta_5 - 8 \alpha_2^2 \beta_6 \alpha_3 \mu_3 + 8 \beta_4 \alpha_1 \alpha_2^2 \mu_4 \\
& + 2 \mu_1 \beta_3 \alpha_3^2 \alpha_2 + 9 \mu_1 \alpha_2 \beta_5 \alpha_1^2 - 20 \beta_3 \mu_3 \alpha_1^3 - 2 \beta_3 \alpha_2 \mu_4 \alpha_3^2 + 8 \alpha_3 \beta_2 \alpha_2 \alpha_1 \mu_4 \\
& + 8 \alpha_2 \beta_6 \alpha_1^2 \mu_3 + 16 \beta_1 \alpha_2^2 \alpha_1 \mu_3 + 16 \beta_4 \alpha_2^3 \mu_3 - 16 \alpha_2^2 \beta_5 \alpha_1 \mu_3 - 8 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_4 \\
& - 9 \alpha_2 \mu_4 \alpha_1^2 \beta_5) x_1 x_3 x_2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} (-4 \alpha_3^2 \mu_4 \beta_3 \alpha_1 \\
& - 4 \beta_6 \alpha_2 \mu_4 \alpha_3^2 + 4 \alpha_2 \alpha_3 \mu_4 \beta_1 \alpha_1 - 4 \alpha_2 \alpha_3 \mu_4 \beta_5 \alpha_1 + 4 \mu_1 \beta_5 \alpha_3 \alpha_2 \alpha_1 - 10 \mu_2 \beta_4 \alpha_1^3 \\
& - 8 \mu_2 \alpha_3^2 \beta_2 \alpha_1 + 8 \mu_2 \beta_6 \alpha_3^2 \alpha_1 - 4 \mu_2 \beta_5 \alpha_3^2 \alpha_2 + 4 \mu_2 \beta_5 \alpha_3 \alpha_1^2 - 8 \mu_2 \beta_3 \alpha_3^3 \\
& - 18 \mu_2 \beta_4 \alpha_2 \alpha_3 \alpha_1 - 13 \beta_1 \mu_2 \alpha_3 \alpha_1^2 + 21 \beta_1 \alpha_2 \mu_3 \alpha_1^2 + 5 \beta_1 \alpha_2^2 \mu_3 \alpha_3 - 5 \beta_1 \mu_2 \alpha_3^2 \alpha_2 \\
& - 4 \mu_1 \alpha_3 \beta_1 \alpha_2 \alpha_1 - 12 \alpha_2 \beta_5 \alpha_1^2 \mu_3 - 8 \beta_3 \alpha_3 \alpha_1^2 \mu_3 + 5 \mu_1 \alpha_3^2 \beta_2 \alpha_2 + 4 \mu_1 \beta_3 \alpha_3^2 \alpha_1 \\
& + 5 \mu_1 \beta_4 \alpha_1^2 \alpha_2 + \mu_1 \beta_4 \alpha_2^2 \alpha_3 + 9 \mu_1 \beta_2 \alpha_1^2 \alpha_3 + 4 \mu_1 \alpha_2 \beta_6 \alpha_3^2 - 9 \beta_2 \mu_4 \alpha_3 \alpha_1^2 \\
& - 18 \beta_2 \mu_3 \alpha_1^3 - 5 \beta_2 \alpha_2 \mu_4 \alpha_3^2 - 5 \alpha_2 \mu_4 \alpha_1^2 \beta_4 - \alpha_2^2 \mu_4 \alpha_3 \beta_4 + 4 \beta_5 \alpha_3 \alpha_2^2 \mu_3 \\
& + 16 \beta_4 \alpha_1 \alpha_2^2 \mu_3 + 8 \beta_3 \alpha_3^2 \alpha_2 \mu_3 - 16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_3 - 2 \beta_2 \alpha_2 \mu_3 \alpha_3 \alpha_1) x_1^2 x_3 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} (-16 \beta_4 \alpha_2^3 \beta_5 - 4 \beta_2 \beta_3 \alpha_3^2 \alpha_2 - 44 \beta_2 \beta_3 \alpha_3 \alpha_1^2 \\
& - 8 \beta_2 \alpha_2 \beta_6 \alpha_3 \alpha_1 - 8 \beta_2^2 \alpha_3 \alpha_2 \alpha_1 + 20 \beta_2 \alpha_2 \beta_5 \alpha_1^2 - 8 \beta_3 \beta_6 \alpha_3^2 \alpha_2 - 48 \beta_3^2 \alpha_3^2 \alpha_1 \\
& - 32 \beta_1 \alpha_2^2 \beta_5 \alpha_1 + 16 \beta_1^2 \alpha_2^2 \alpha_1 - 4 \beta_2 \beta_5 \alpha_3 \alpha_2^2 - 32 \beta_1 \beta_3 \alpha_1^3 - 8 \beta_1 \beta_2 \alpha_1^2 \alpha_2 \\
& + 16 \beta_1 \beta_4 \alpha_2^3 + 16 \beta_1 \alpha_3 \beta_2 \alpha_2^2 - 16 \beta_1 \alpha_2^2 \beta_6 \alpha_3 - 16 \beta_1 \alpha_2 \beta_6 \alpha_1^2 - 8 \beta_2 \beta_4 \alpha_1 \alpha_2^2 \\
& + 4 \alpha_2^2 \beta_6 \alpha_3 \beta_5 - 16 \beta_4 \alpha_1 \alpha_2^2 \beta_6 + 56 \beta_3 \beta_6 \alpha_1^2 \alpha_3 + 32 \beta_3 \beta_5 \alpha_1^3 + 24 \beta_3 \beta_4 \alpha_2^2 \alpha_3 \\
& - 24 \beta_3 \beta_4 \alpha_1^2 \alpha_2 + 16 \alpha_2 \beta_6^2 \alpha_3 \alpha_1 + 16 \alpha_2^2 \beta_5^2 \alpha_1 + 4 \alpha_2 \beta_6 \alpha_1^2 \beta_5) x_1 x_2^2 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} (-16 \beta_1 \alpha_2 \beta_6 \alpha_3 \alpha_1 + 16 \beta_1 \alpha_3 \beta_2 \alpha_2 \alpha_1 \\
& + 16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \beta_5 - 16 \beta_4^2 \alpha_2^3 + 4 \beta_2 \beta_4 \alpha_2^2 \alpha_3 - 16 \beta_2 \beta_5 \alpha_3 \alpha_2 \alpha_1 - 24 \beta_2^2 \alpha_1^2 \alpha_3 \\
& - 8 \beta_2 \beta_3 \alpha_3^2 \alpha_1 + 16 \beta_3 \beta_4 \alpha_1^3 - 8 \alpha_2 \beta_6 \alpha_1^2 \beta_4 + 12 \alpha_2 \beta_5^2 \alpha_1^2 + 48 \beta_3 \beta_4 \alpha_2 \alpha_3 \alpha_1 \\
& - 4 \beta_1 \beta_5 \alpha_3 \alpha_2^2 - 16 \beta_3 \beta_6 \alpha_3^2 \alpha_1 + 24 \beta_2 \beta_6 \alpha_1^2 \alpha_3 + 24 \beta_2 \beta_5 \alpha_1^3 - 24 \beta_1 \beta_2 \alpha_1^3 \\
& - 4 \beta_5^2 \alpha_3 \alpha_2^2 - 36 \beta_1 \alpha_2 \beta_5 \alpha_1^2 + 24 \beta_1^2 \alpha_1^2 \alpha_2 + 16 \beta_3^2 \alpha_3^3 + 8 \alpha_2^2 \beta_6 \alpha_3 \beta_4 \\
& - 4 \beta_2 \beta_4 \alpha_1^2 \alpha_2 + 8 \beta_1^2 \alpha_3 \alpha_2^2) x_1^2 x_2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} ( \\
& 32 \beta_3 \alpha_2 \beta_6 \alpha_3 \alpha_1 - 4 \beta_2^2 \alpha_1^2 \alpha_2 + 36 \beta_3 \beta_5 \alpha_1^2 \alpha_2 + 16 \beta_3^2 \alpha_3 \alpha_1^2 - 12 \beta_2 \alpha_2^2 \beta_6 \alpha_3 \\
& + 4 \beta_3 \beta_5 \alpha_3 \alpha_2^2 + 8 \alpha_2^2 \beta_6^2 \alpha_3 - 16 \beta_4 \alpha_2^3 \beta_6 + 8 \beta_3 \alpha_3 \beta_1 \alpha_2^2 + 8 \beta_2 \beta_1 \alpha_2^2 \alpha_1 \\
& - 16 \beta_3^2 \alpha_3^2 \alpha_2 - 32 \beta_2 \beta_3 \alpha_3 \alpha_1 \alpha_2 - 8 \alpha_2 \beta_6^2 \alpha_1^2 + 4 \beta_2^2 \alpha_3 \alpha_2^2 - 16 \beta_1 \alpha_2^2 \alpha_1 \beta_6
\end{aligned}$$

$$\begin{aligned}
& -8\beta_2\alpha_2^2\beta_5\alpha_1 + 16\alpha_2^2\beta_5\alpha_1\beta_6 + 12\beta_2\alpha_2\beta_6\alpha_1^2 - 24\beta_1\alpha_1^2\alpha_2\beta_3 - 32\beta_3\beta_4\alpha_1\alpha_2^2 \\
& + 8\beta_2\beta_3\alpha_1^3 + 8\beta_2\beta_4\alpha_2^3 - 16\beta_3\beta_6\alpha_1^3)x_2^3 / (\alpha_2^2\alpha_3^2 + 2\alpha_1^2\alpha_2\alpha_3 + \alpha_1^4) + \frac{1}{24} ( \\
& 36\beta_1\alpha_2\beta_5\alpha_1^2 + 24\beta_2^2\alpha_1^2\alpha_3 - 16\beta_3\beta_4\alpha_1^3 - 16\alpha_2\beta_6\alpha_3\alpha_1\beta_5 - 16\beta_3^2\alpha_3^3 \\
& - 8\beta_1^2\alpha_3\alpha_2^2 + 16\beta_3\beta_6\alpha_3^2\alpha_1 - 12\alpha_2\beta_5^2\alpha_1^2 + 16\beta_4^2\alpha_2^3 - 4\beta_2\beta_4\alpha_2^2\alpha_3 \\
& - 16\beta_1\alpha_3\beta_2\alpha_2\alpha_1 + 16\beta_1\alpha_2\beta_6\alpha_3\alpha_1 + 8\alpha_2\beta_6\alpha_1^2\beta_4 + 8\beta_2\beta_3\alpha_3^2\alpha_1 \\
& + 16\beta_2\beta_5\alpha_3\alpha_2\alpha_1 + 4\beta_5^2\alpha_3\alpha_2^2 + 4\beta_1\beta_5\alpha_3\alpha_2^2 - 24\beta_1^2\alpha_1^2\alpha_2 - 48\beta_3\beta_4\alpha_2\alpha_3\alpha_1 \\
& + 24\beta_1\beta_2\alpha_1^3 - 24\beta_2\beta_5\alpha_1^3 + 4\beta_2\beta_4\alpha_1^2\alpha_2 - 24\beta_2\beta_6\alpha_1^2\alpha_3 - 8\alpha_2^2\beta_6\alpha_3\beta_4)x_1^2x_2 \\
& / (\alpha_2^2\alpha_3^2 + 2\alpha_1^2\alpha_2\alpha_3 + \alpha_1^4) + \frac{1}{12} (6\mu_2\alpha_1^3\mu_4 + 3\alpha_2\mu_4^2\alpha_1^2 - 6\mu_1\alpha_2^2\mu_4\alpha_3 \\
& + 6\mu_2\alpha_3\alpha_1\alpha_2\mu_4 + 6\mu_2\mu_3\alpha_1^2\alpha_2 - 6\mu_1\alpha_2\mu_4\alpha_1^2 + 3\mu_1^2\alpha_1^2\alpha_2 + 6\mu_2\alpha_2^2\mu_3\alpha_3 \\
& + 3\mu_1^2\alpha_2^2\alpha_3 + 3\alpha_2^2\mu_4^2\alpha_3 - 6\mu_1\mu_2\alpha_1^3 - 6\mu_2^2\alpha_3\alpha_1^2 - 6\mu_2^2\alpha_3^2\alpha_2 \\
& - 6\mu_1\mu_2\alpha_3\alpha_1\alpha_2)x_2x_3^2 / (\alpha_2^2\alpha_3^2 + 2\alpha_1^2\alpha_2\alpha_3 + \alpha_1^4) + \frac{1}{12} (-3\mu_1\mu_2\alpha_3\alpha_1^2 \\
& + 3\alpha_2^2\mu_4\alpha_3\mu_3 - 3\mu_1\alpha_2\mu_3\alpha_1^2 - 3\mu_1\alpha_2^2\mu_3\alpha_3 + 3\alpha_2\mu_4\alpha_1^2\mu_3 + 12\mu_2\mu_3\alpha_1^3 \\
& + 3\mu_2\alpha_2\mu_4\alpha_3^2 + 12\mu_2\alpha_2\mu_3\alpha_3\alpha_1 + 3\mu_2\mu_4\alpha_3\alpha_1^2 - 3\mu_1\mu_2\alpha_3^2\alpha_2)x_1x_3^2 / ( \\
& \alpha_2^2\alpha_3^2 + 2\alpha_1^2\alpha_2\alpha_3 + \alpha_1^4) + \frac{1}{24} (16\beta_4\alpha_2^3\beta_5 + 4\beta_2\beta_3\alpha_3^2\alpha_2 + 44\beta_2\beta_3\alpha_3\alpha_1^2 \\
& + 8\beta_2\alpha_2\beta_6\alpha_3\alpha_1 + 8\beta_2^2\alpha_3\alpha_2\alpha_1 - 20\beta_2\alpha_2\beta_5\alpha_1^2 + 8\beta_3\beta_6\alpha_3^2\alpha_2 + 48\beta_3^2\alpha_3^2\alpha_1 \\
& + 32\beta_1\alpha_2^2\beta_5\alpha_1 - 16\beta_1^2\alpha_2^2\alpha_1 + 4\beta_2\beta_5\alpha_3\alpha_2^2 + 32\beta_1\beta_3\alpha_1^3 + 8\beta_1\beta_2\alpha_1^2\alpha_2 \\
& - 16\beta_1\beta_4\alpha_2^3 - 16\beta_1\alpha_3\beta_2\alpha_2^2 + 16\beta_1\alpha_2^2\beta_6\alpha_3 + 16\beta_1\alpha_2\beta_6\alpha_1^2 + 8\beta_2\beta_4\alpha_1\alpha_2^2 \\
& - 4\alpha_2^2\beta_6\alpha_3\beta_5 + 16\beta_4\alpha_1\alpha_2^2\beta_6 - 56\beta_3\beta_6\alpha_1^2\alpha_3 - 32\beta_3\beta_5\alpha_1^3 - 24\beta_3\beta_4\alpha_2^2\alpha_3 \\
& + 24\beta_3\beta_4\alpha_1^2\alpha_2 - 16\alpha_2\beta_6^2\alpha_3\alpha_1 - 16\alpha_2^2\beta_5^2\alpha_1 - 4\alpha_2\beta_6\alpha_1^2\beta_5)x_1x_2^2 / ( \\
& \alpha_2^2\alpha_3^2 + 2\alpha_1^2\alpha_2\alpha_3 + \alpha_1^4) + \frac{1}{12} (6\beta_1\mu_1\alpha_2^2\alpha_3 - 16\mu_2\beta_4\alpha_1^2\alpha_2 - 6\mu_1\beta_3\alpha_3\alpha_1^2 \\
& - 3\mu_1\beta_5\alpha_3\alpha_2^2 + 12\beta_2\alpha_1^3\mu_4 - 5\beta_2\alpha_2^2\mu_3\alpha_3 - 7\beta_2\mu_2\alpha_3^2\alpha_2 - 39\beta_2\mu_2\alpha_3\alpha_1^2 \\
& + 11\beta_2\alpha_2\mu_3\alpha_1^2 + 14\beta_3\mu_4\alpha_3\alpha_1^2 - 32\mu_2\beta_3\alpha_3^2\alpha_1 - 8\mu_2\beta_6\alpha_3^2\alpha_2 \\
& + 36\beta_3\alpha_2\mu_3\alpha_3\alpha_1 + 18\mu_2\beta_5\alpha_1^3 + 24\mu_2\beta_6\alpha_1^2\alpha_3 + 16\mu_2\beta_4\alpha_2^2\alpha_3 \\
& - 14\mu_2\beta_5\alpha_3\alpha_1\alpha_2 + 6\beta_1\mu_1\alpha_1^2\alpha_2 - 14\beta_1\alpha_2^2\mu_4\alpha_3 - 4\beta_1\mu_2\alpha_3\alpha_1\alpha_2 \\
& - 30\beta_1\alpha_2\mu_4\alpha_1^2 - 36\beta_1\mu_2\alpha_1^3 - \alpha_2^2\mu_4\alpha_3\beta_5 + 8\alpha_2^2\beta_6\alpha_3\mu_3 - 16\beta_4\alpha_1\alpha_2^2\mu_4 \\
& - 6\mu_1\beta_3\alpha_3^2\alpha_2 - 3\mu_1\alpha_2\beta_5\alpha_1^2 + 20\beta_3\mu_3\alpha_1^3 - 2\beta_3\alpha_2\mu_4\alpha_3^2 - 4\alpha_3\beta_2\alpha_2\alpha_1\mu_4
\end{aligned}$$

$$\begin{aligned}
& -8 \alpha_2 \beta_6 \alpha_1^2 \mu_3 - 16 \beta_1 \alpha_2^2 \alpha_1 \mu_3 - 16 \beta_4 \alpha_2^3 \mu_3 + 16 \alpha_2^2 \beta_5 \alpha_1 \mu_3 + 16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_4 \\
& + 15 \alpha_2 \mu_4 \alpha_1^2 \beta_5) x_1 x_3 x_2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} (-36 \beta_3 \beta_5 \alpha_1^2 \alpha_2 \\
& + 32 \beta_2 \beta_3 \alpha_3 \alpha_1 \alpha_2 - 12 \beta_2 \alpha_2 \beta_6 \alpha_1^2 - 4 \beta_3 \beta_5 \alpha_3 \alpha_2^2 + 8 \beta_2 \alpha_2^2 \beta_5 \alpha_1 - 8 \beta_2 \beta_3 \alpha_1^3 \\
& + 12 \beta_2 \alpha_2^2 \beta_6 \alpha_3 + 16 \beta_3 \beta_6 \alpha_1^3 + 16 \beta_4 \alpha_2^3 \beta_6 - 16 \beta_3^2 \alpha_3 \alpha_1^2 + 32 \beta_3 \beta_4 \alpha_1 \alpha_2^2 \\
& - 8 \alpha_2^2 \beta_6^2 \alpha_3 - 8 \beta_3 \alpha_3 \beta_1 \alpha_2^2 + 16 \beta_3^2 \alpha_3^2 \alpha_2 - 8 \beta_2 \beta_1 \alpha_2^2 \alpha_1 - 16 \alpha_2^2 \beta_5 \alpha_1 \beta_6 \\
& - 32 \beta_3 \alpha_2 \beta_6 \alpha_3 \alpha_1 + 24 \beta_1 \alpha_1^2 \alpha_2 \beta_3 - 8 \beta_2 \beta_4 \alpha_2^3 + 4 \beta_2^2 \alpha_1^2 \alpha_2 - 4 \beta_2^2 \alpha_3 \alpha_2^2 \\
& + 8 \alpha_2 \beta_6^2 \alpha_1^2 + 16 \beta_1 \alpha_2^2 \alpha_1 \beta_6) x_2^3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} ( \\
& 4 \beta_2 \beta_5 \alpha_3 \alpha_1^2 - 8 \beta_2^2 \alpha_3^2 \alpha_1 - 4 \beta_1 \beta_2 \alpha_1^2 \alpha_3 - 4 \beta_2 \beta_5 \alpha_3^2 \alpha_2 + 16 \beta_4^2 \alpha_1 \alpha_2^2 \\
& + 4 \beta_1 \alpha_3^2 \beta_2 \alpha_2 - 16 \beta_2 \beta_4 \alpha_1^3 - 12 \alpha_2 \beta_5 \alpha_1^2 \beta_4 - 8 \beta_3 \alpha_3 \alpha_1^2 \beta_4 + 8 \beta_1 \beta_4 \alpha_2^2 \alpha_3 \\
& - 8 \beta_2 \beta_3 \alpha_3^3 + 8 \beta_2 \beta_6 \alpha_3^2 \alpha_1 - 16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \beta_4 - 8 \beta_2 \beta_4 \alpha_2 \alpha_3 \alpha_1 + 8 \beta_3 \alpha_3^2 \alpha_2 \beta_4 \\
& + 4 \beta_5 \alpha_3 \alpha_2^2 \beta_4 + 24 \beta_1 \beta_4 \alpha_1^2 \alpha_2) x_1^3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \gamma_4 x_2^3 \\
& + \gamma_2 x_1^2 x_2 + \gamma_1 x_1^3 + \frac{1}{12} (-8 \mu_1 \beta_5 \alpha_3 \alpha_2 \alpha_1 + 10 \mu_2 \beta_4 \alpha_1^3 + 8 \mu_2 \alpha_3^2 \beta_2 \alpha_1 \\
& - 8 \mu_2 \beta_6 \alpha_3^2 \alpha_1 + 4 \mu_2 \beta_5 \alpha_3^2 \alpha_2 - 4 \mu_2 \beta_5 \alpha_3 \alpha_1^2 + 8 \mu_2 \beta_3 \alpha_3^3 + 18 \mu_2 \beta_4 \alpha_2 \alpha_3 \alpha_1 \\
& + 7 \beta_1 \mu_2 \alpha_3 \alpha_1^2 - 27 \beta_1 \alpha_2 \mu_3 \alpha_1^2 - 11 \beta_1 \alpha_2^2 \mu_3 \alpha_3 - \beta_1 \mu_2 \alpha_3^2 \alpha_2 - 4 \mu_1 \alpha_3 \beta_1 \alpha_2 \alpha_1 \\
& + 12 \alpha_2 \beta_5 \alpha_1^2 \mu_3 + 8 \beta_3 \alpha_3 \alpha_1^2 \mu_3 - 7 \mu_1 \alpha_3^2 \beta_2 \alpha_2 - 12 \mu_1 \beta_1 \alpha_1^3 - 8 \mu_1 \beta_3 \alpha_3^2 \alpha_1 \\
& - 7 \mu_1 \beta_4 \alpha_1^2 \alpha_2 + \mu_1 \beta_4 \alpha_2^2 \alpha_3 - 15 \mu_1 \beta_2 \alpha_1^2 \alpha_3 - 8 \mu_1 \alpha_2 \beta_6 \alpha_3^2 + 3 \beta_2 \mu_4 \alpha_3 \alpha_1^2 \\
& + 18 \beta_2 \mu_3 \alpha_1^3 + 3 \beta_2 \alpha_2 \mu_4 \alpha_3^2 + 3 \alpha_2 \mu_4 \alpha_1^2 \beta_4 + 3 \alpha_2^2 \mu_4 \alpha_3 \beta_4 - 4 \beta_5 \alpha_3 \alpha_2^2 \mu_3 \\
& - 16 \beta_4 \alpha_1 \alpha_2^2 \mu_3 - 8 \beta_3 \alpha_3^2 \alpha_2 \mu_3 + 16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_3 + 2 \beta_2 \alpha_2 \mu_3 \alpha_3 \alpha_1) x_1^2 x_3 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4), \frac{1}{24} (-8 \alpha_2^2 \beta_5^2 \alpha_1 - 4 \beta_2 \beta_3 \alpha_3^2 \alpha_2 + 8 \beta_3 \beta_4 \alpha_1^2 \alpha_2 \\
& - 8 \beta_3 \beta_6 \alpha_3^2 \alpha_2 + 12 \beta_2 \beta_3 \alpha_3 \alpha_1^2 - 8 \beta_5 \beta_3 \alpha_3 \alpha_1 \alpha_2 - 24 \beta_3 \beta_6 \alpha_1^2 \alpha_3 + 4 \alpha_2 \beta_6 \alpha_1^2 \beta_5 \\
& + 8 \beta_1 \alpha_2^2 \beta_5 \alpha_1 - 8 \beta_3 \beta_4 \alpha_2^2 \alpha_3 - 4 \beta_2 \alpha_2 \beta_5 \alpha_1^2 + 16 \beta_3^2 \alpha_3^2 \alpha_1 + 8 \beta_4 \alpha_2^3 \beta_5 \\
& + 4 \beta_2 \beta_5 \alpha_3 \alpha_2^2 - 16 \alpha_3 \beta_1 \alpha_1 \alpha_2 \beta_3 - 4 \alpha_2^2 \beta_6 \alpha_3 \beta_5 - 16 \beta_3 \beta_5 \alpha_1^3) x_2^3 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \rho_4 x_2 x_3^2 + \rho_3 x_1 x_3^2 + \nu_6 x_2^2 x_3 + \nu_4 x_1^2 x_3 + \gamma_7 x_1 x_2^2 \\
& + \gamma_6 x_1^2 x_2 + \frac{1}{24} (4 \beta_5^2 \alpha_3^2 \alpha_2 + 8 \beta_5 \beta_3 \alpha_3^3 - 16 \beta_4^2 \alpha_2^2 \alpha_3 - 32 \beta_4 \alpha_3 \beta_1 \alpha_2 \alpha_1 \\
& - 8 \beta_5 \beta_4 \alpha_1^3 - 12 \beta_5 \alpha_3^2 \beta_1 \alpha_2 + 8 \beta_4 \alpha_2 \beta_6 \alpha_3^2 + 8 \alpha_3^2 \beta_1^2 \alpha_2 - 4 \beta_5^2 \alpha_3 \alpha_1^2 \\
& + 4 \beta_4 \alpha_3^2 \beta_2 \alpha_2 - 8 \alpha_3 \beta_1^2 \alpha_1^2 - 24 \beta_6 \alpha_1^2 \alpha_3 \beta_4 + 16 \beta_4^2 \alpha_1^2 \alpha_2 + 32 \beta_4 \beta_3 \alpha_3^2 \alpha_1
\end{aligned}$$



$$\begin{aligned}
& + 36 \beta_4 \beta_2 \alpha_1^2 \alpha_3 + 16 \beta_6 \alpha_3^2 \alpha_1 \beta_1 - 16 \alpha_3^2 \beta_2 \alpha_1 \beta_1 - 8 \beta_5 \beta_6 \alpha_3^2 \alpha_1 + 12 \beta_5 \alpha_3 \beta_1 \alpha_1^2 \\
& + 8 \beta_5 \alpha_3^2 \beta_2 \alpha_1 - 16 \beta_3 \alpha_3^3 \beta_1 + 32 \beta_4 \beta_5 \alpha_3 \alpha_2 \alpha_1 + 16 \beta_4 \beta_1 \alpha_1^3) x_1^3 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} (-3 \alpha_2^2 \mu_4 \alpha_3 \mu_3 + 3 \mu_1 \alpha_2^2 \mu_3 \alpha_3 + 3 \mu_1 \alpha_2 \mu_3 \alpha_1^2 \\
& - 12 \mu_2 \mu_3 \alpha_1^3 + 3 \mu_1 \mu_2 \alpha_3^2 \alpha_2 - 12 \mu_2 \alpha_2 \mu_3 \alpha_3 \alpha_1 - 3 \mu_2 \alpha_2 \mu_4 \alpha_3^2 - 3 \mu_2 \mu_4 \alpha_3 \alpha_1^2 \\
& - 3 \alpha_2 \mu_4 \alpha_1^2 \mu_3 + 3 \mu_1 \mu_2 \alpha_3 \alpha_1^2) x_2 x_3^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) - \frac{1}{24} ( \\
& 6 \mu_1 \alpha_2 \mu_3 \alpha_1^2 + 6 \alpha_2 \alpha_3 \alpha_1 \mu_4 \mu_1 + 6 \mu_1 \mu_2 \alpha_3 \alpha_1^2 + 3 \alpha_2^2 \alpha_1 \mu_3^2 - 3 \alpha_2 \alpha_3 \alpha_1 \mu_1^2 \\
& - 12 \mu_2 \mu_3 \alpha_1^3 - 6 \mu_2 \mu_4 \alpha_3 \alpha_1^2 + 3 \alpha_3^2 \alpha_1 \mu_2^2 - 6 \alpha_2 \mu_4 \alpha_1^2 \mu_3 - 3 \alpha_2 \alpha_3 \alpha_1 \mu_4^2 \\
& - 6 \mu_2 \alpha_2 \mu_3 \alpha_3 \alpha_1) x_2 x_3^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) - \frac{1}{24} (-3 \mu_3^2 \alpha_2^2 \alpha_3 \\
& - 3 \alpha_3^3 \mu_2^2 + 6 \alpha_3^2 \alpha_1 \mu_4 \mu_2 + 6 \mu_3 \mu_2 \alpha_3^2 \alpha_2 - 6 \alpha_3^2 \alpha_1 \mu_1 \mu_2 + 12 \mu_3 \mu_2 \alpha_3 \alpha_1^2 \\
& + 3 \mu_4^2 \alpha_2 \alpha_3^2 + 3 \mu_1^2 \alpha_2 \alpha_3^2 - 6 \mu_4 \mu_1 \alpha_2 \alpha_3^2 + 6 \mu_4 \alpha_2 \mu_3 \alpha_3 \alpha_1 - 6 \alpha_2 \mu_3 \alpha_3 \alpha_1 \mu_1) x_1 \\
& x_3^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} (3 \mu_1^2 \alpha_2 \alpha_3^2 + 6 \mu_4 \mu_3 \alpha_1^3 + 3 \mu_4^2 \alpha_3 \alpha_1^2 \\
& + 3 \alpha_3 \mu_1^2 \alpha_1^2 + 6 \mu_3 \mu_2 \alpha_3^2 \alpha_2 - 6 \mu_3 \alpha_1^3 \mu_1 - 6 \alpha_2 \mu_3 \alpha_3 \alpha_1 \mu_1 + 6 \mu_3 \mu_2 \alpha_3 \alpha_1^2 \\
& - 6 \mu_3^2 \alpha_2 \alpha_1^2 - 6 \mu_4 \mu_1 \alpha_2 \alpha_3^2 + 3 \mu_4^2 \alpha_2 \alpha_3^2 - 6 \mu_4 \alpha_3 \mu_1 \alpha_1^2 + 6 \mu_4 \alpha_2 \mu_3 \alpha_3 \alpha_1 \\
& - 6 \mu_3^2 \alpha_2^2 \alpha_3) x_1 x_3^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + v_5 x_1 x_2 x_3 - \frac{1}{24} ( \\
& - 8 \alpha_3^2 \mu_4 \beta_3 \alpha_1 - 18 \beta_6 \alpha_3 \mu_1 \alpha_1^2 + 18 \beta_6 \mu_4 \alpha_3 \alpha_1^2 + 10 \beta_6 \alpha_2 \mu_4 \alpha_3^2 + 8 \alpha_2 \alpha_3 \mu_4 \beta_1 \alpha_1 \\
& - 8 \alpha_2 \alpha_3 \mu_4 \beta_5 \alpha_1 + 36 \beta_6 \mu_3 \alpha_1^3 + 8 \mu_1 \beta_5 \alpha_3 \alpha_2 \alpha_1 - 20 \mu_2 \beta_4 \alpha_1^3 - 16 \mu_2 \alpha_3^2 \beta_2 \alpha_1 \\
& + 16 \mu_2 \beta_6 \alpha_3^2 \alpha_1 + \mu_2 \beta_5 \alpha_3^2 \alpha_2 + 17 \mu_2 \beta_5 \alpha_3 \alpha_1^2 - 16 \mu_2 \beta_3 \alpha_3^3 - 36 \mu_2 \beta_4 \alpha_2 \alpha_3 \alpha_1 \\
& - 8 \beta_1 \mu_2 \alpha_3 \alpha_1^2 + 24 \beta_1 \alpha_2 \mu_3 \alpha_1^2 - 8 \beta_1 \alpha_2^2 \mu_3 \alpha_3 + 8 \beta_1 \mu_2 \alpha_3^2 \alpha_2 - 8 \mu_1 \alpha_3 \beta_1 \alpha_2 \alpha_1 \\
& - 33 \alpha_2 \beta_5 \alpha_1^2 \mu_3 - 16 \beta_3 \alpha_3 \alpha_1^2 \mu_3 + \mu_1 \alpha_3^2 \beta_2 \alpha_2 + 8 \mu_1 \beta_3 \alpha_3^2 \alpha_1 + 10 \mu_1 \beta_4 \alpha_1^2 \alpha_2 \\
& + 2 \mu_1 \beta_4 \alpha_2^2 \alpha_3 + 9 \mu_1 \beta_2 \alpha_1^2 \alpha_3 - 10 \mu_1 \alpha_2 \beta_6 \alpha_3^2 - 9 \beta_2 \mu_4 \alpha_3 \alpha_1^2 - 18 \beta_2 \mu_3 \alpha_1^3 \\
& - \beta_2 \alpha_2 \mu_4 \alpha_3^2 - 10 \alpha_2 \mu_4 \alpha_1^2 \beta_4 - 2 \alpha_2^2 \mu_4 \alpha_3 \beta_4 - \beta_5 \alpha_3 \alpha_2^2 \mu_3 + 32 \beta_4 \alpha_1 \alpha_2^2 \mu_3 \\
& + 16 \beta_3 \alpha_3^2 \alpha_2 \mu_3 + 4 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_3 + 14 \beta_2 \alpha_2 \mu_3 \alpha_3 \alpha_1) x_1 x_3 x_2 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) - \frac{1}{24} (-4 \alpha_2^2 \mu_1 \beta_4 \alpha_1 + 4 \alpha_2 \alpha_3 \mu_1 \beta_6 \alpha_1 - 4 \alpha_2 \alpha_3 \mu_1 \beta_2 \alpha_1 \\
& + 4 \beta_1 \mu_1 \alpha_2^2 \alpha_3 + 8 \mu_2 \beta_4 \alpha_1^2 \alpha_2 + 5 \mu_1 \beta_3 \alpha_3 \alpha_1^2 + 5 \mu_1 \beta_5 \alpha_3 \alpha_2^2 + 4 \beta_2 \alpha_2^2 \mu_3 \alpha_3 \\
& - 4 \beta_2 \mu_2 \alpha_3^2 \alpha_2 + 12 \beta_2 \mu_2 \alpha_3 \alpha_1^2 - 4 \beta_2 \alpha_2 \mu_3 \alpha_1^2 - 5 \beta_3 \mu_4 \alpha_3 \alpha_1^2 + 16 \mu_2 \beta_3 \alpha_3^2 \alpha_1
\end{aligned}$$

$$\begin{aligned}
& -5 \mu_2 \beta_6 \alpha_3^2 \alpha_2 - 18 \beta_3 \alpha_2 \mu_3 \alpha_3 \alpha_1 - 18 \mu_2 \beta_5 \alpha_1^3 - 21 \mu_2 \beta_6 \alpha_1^2 \alpha_3 - 8 \mu_2 \beta_4 \alpha_2^2 \alpha_3 \\
& - 2 \mu_2 \beta_5 \alpha_3 \alpha_1 \alpha_2 - 4 \beta_1 \alpha_2^2 \mu_4 \alpha_3 - 16 \beta_1 \mu_2 \alpha_3 \alpha_1 \alpha_2 - 5 \alpha_2^2 \mu_4 \alpha_3 \beta_5 + 5 \alpha_2^2 \beta_6 \alpha_3 \mu_3 \\
& + 4 \beta_4 \alpha_1 \alpha_2^2 \mu_4 + \mu_1 \beta_3 \alpha_3^2 \alpha_2 + 9 \mu_1 \alpha_2 \beta_5 \alpha_1^2 - 10 \beta_3 \mu_3 \alpha_1^3 - \beta_3 \alpha_2 \mu_4 \alpha_3^2 \\
& + 4 \alpha_3 \beta_2 \alpha_2 \alpha_1 \mu_4 + 13 \alpha_2 \beta_6 \alpha_1^2 \mu_3 + 8 \beta_1 \alpha_2^2 \alpha_1 \mu_3 + 8 \beta_4 \alpha_2^3 \mu_3 - 8 \alpha_2^2 \beta_5 \alpha_1 \mu_3 \\
& - 4 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_4 - 9 \alpha_2 \mu_4 \alpha_1^2 \beta_5) x_2^2 x_3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} ( \\
& -30 \beta_6 \alpha_3 \mu_1 \alpha_1^2 + 6 \beta_6 \mu_4 \alpha_3 \alpha_1^2 + 6 \beta_6 \alpha_2 \mu_4 \alpha_3^2 + 36 \beta_6 \mu_3 \alpha_1^3 - 12 \beta_5 \alpha_1^3 \mu_1 \\
& + 4 \mu_1 \beta_5 \alpha_3 \alpha_2 \alpha_1 - 20 \mu_2 \beta_4 \alpha_1^3 - 16 \mu_2 \alpha_3^2 \beta_2 \alpha_1 + 16 \mu_2 \beta_6 \alpha_3^2 \alpha_1 - 5 \mu_2 \beta_5 \alpha_3^2 \alpha_2 \\
& + 11 \mu_2 \beta_5 \alpha_3 \alpha_1^2 - 16 \mu_2 \beta_3 \alpha_3^3 - 36 \mu_2 \beta_4 \alpha_2 \alpha_3 \alpha_1 - 8 \beta_1 \mu_2 \alpha_3 \alpha_1^2 + 24 \beta_1 \alpha_2 \mu_3 \alpha_1^2 \\
& - 8 \beta_1 \alpha_2^2 \mu_3 \alpha_3 + 8 \beta_1 \mu_2 \alpha_3^2 \alpha_2 - 16 \mu_1 \alpha_3 \beta_1 \alpha_2 \alpha_1 - 39 \alpha_2 \beta_5 \alpha_1^2 \mu_3 - 16 \beta_3 \alpha_3 \alpha_1^2 \mu_3 \\
& - \mu_1 \alpha_3^2 \beta_2 \alpha_2 + 16 \mu_1 \beta_3 \alpha_3^2 \alpha_1 + 14 \mu_1 \beta_4 \alpha_1^2 \alpha_2 - 2 \mu_1 \beta_4 \alpha_2^2 \alpha_3 + 15 \mu_1 \beta_2 \alpha_1^2 \alpha_3 \\
& - 14 \mu_1 \alpha_2 \beta_6 \alpha_3^2 - 3 \beta_2 \mu_4 \alpha_3 \alpha_1^2 - 18 \beta_2 \mu_3 \alpha_1^3 - 3 \beta_2 \alpha_2 \mu_4 \alpha_3^2 - 6 \alpha_2 \mu_4 \alpha_1^2 \beta_4 \\
& - 6 \alpha_2^2 \mu_4 \alpha_3 \beta_4 - 7 \beta_5 \alpha_3 \alpha_2^2 \mu_3 + 32 \beta_4 \alpha_1 \alpha_2^2 \mu_3 + 16 \beta_3 \alpha_3^2 \alpha_2 \mu_3 + 4 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_3 \\
& + 14 \beta_2 \alpha_2 \mu_3 \alpha_3 \alpha_1) x_1 x_3 x_2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} (12 \mu_4 \beta_6 \alpha_1^3 \\
& + 8 \mu_2 \beta_4 \alpha_1^2 \alpha_2 + 3 \mu_1 \beta_3 \alpha_3 \alpha_1^2 + 3 \mu_1 \beta_5 \alpha_3 \alpha_2^2 + 4 \beta_2 \alpha_2^2 \mu_3 \alpha_3 - 4 \beta_2 \mu_2 \alpha_3^2 \alpha_2 \\
& + 12 \beta_2 \mu_2 \alpha_3 \alpha_1^2 - 4 \beta_2 \alpha_2 \mu_3 \alpha_1^2 - 7 \beta_3 \mu_4 \alpha_3 \alpha_1^2 + 16 \mu_2 \beta_3 \alpha_3^2 \alpha_1 - 11 \mu_2 \beta_6 \alpha_3^2 \alpha_2 \\
& - 18 \beta_3 \alpha_2 \mu_3 \alpha_3 \alpha_1 - 18 \mu_2 \beta_5 \alpha_1^3 - 27 \mu_2 \beta_6 \alpha_1^2 \alpha_3 - 8 \mu_2 \beta_4 \alpha_2^2 \alpha_3 - 2 \mu_2 \beta_5 \alpha_3 \alpha_1 \alpha_2 \\
& - 8 \beta_1 \alpha_2^2 \mu_4 \alpha_3 - 16 \beta_1 \mu_2 \alpha_3 \alpha_1 \alpha_2 - 7 \alpha_2^2 \mu_4 \alpha_3 \beta_5 - \alpha_2^2 \beta_6 \alpha_3 \mu_3 + 8 \beta_4 \alpha_1 \alpha_2^2 \mu_4 \\
& + 3 \mu_1 \beta_3 \alpha_3^2 \alpha_2 + 3 \mu_1 \alpha_2 \beta_5 \alpha_1^2 - 10 \beta_3 \mu_3 \alpha_1^3 + \beta_3 \alpha_2 \mu_4 \alpha_3^2 + 8 \alpha_3 \beta_2 \alpha_2 \alpha_1 \mu_4 \\
& + 7 \alpha_2 \beta_6 \alpha_1^2 \mu_3 + 8 \beta_1 \alpha_2^2 \alpha_1 \mu_3 + 8 \beta_4 \alpha_2^3 \mu_3 - 8 \alpha_2^2 \beta_5 \alpha_1 \mu_3 + 4 \alpha_2 \beta_6 \alpha_3 \alpha_1 \mu_4 \\
& - 15 \alpha_2 \mu_4 \alpha_1^2 \beta_5) x_2^2 x_3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{12} (9 \beta_4 \mu_2 \alpha_3^2 \alpha_2 \\
& - 21 \beta_4 \alpha_2^2 \mu_3 \alpha_3 + 3 \beta_4 \alpha_2 \mu_3 \alpha_1^2 + 18 \mu_3 \beta_5 \alpha_3 \alpha_2 \alpha_1 - 18 \mu_3 \alpha_3 \beta_1 \alpha_2 \alpha_1 \\
& + 16 \beta_6 \alpha_3^2 \alpha_1 \mu_1 + 9 \beta_4 \mu_2 \alpha_3 \alpha_1^2 + 11 \mu_1 \alpha_2 \alpha_3^2 \beta_1 + 8 \mu_4 \beta_3 \alpha_3^3 + 4 \mu_4 \beta_4 \alpha_1^3 \\
& + 12 \mu_4 \beta_4 \alpha_2 \alpha_3 \alpha_1 - 8 \mu_4 \beta_6 \alpha_3^2 \alpha_1 + 8 \mu_4 \alpha_3^2 \beta_2 \alpha_1 + \mu_4 \alpha_3 \beta_1 \alpha_1^2 + 6 \mu_3 \beta_1 \alpha_1^3 \\
& - 6 \beta_5 \mu_3 \alpha_1^3 - 16 \beta_3 \alpha_3^3 \mu_1 - 5 \alpha_3 \mu_1 \alpha_1^2 \beta_1 - 7 \mu_4 \alpha_3^2 \beta_1 \alpha_2 - 8 \beta_4 \alpha_1^3 \mu_1 \\
& - 24 \beta_4 \alpha_2 \alpha_3 \alpha_1 \mu_1 - 11 \beta_5 \mu_1 \alpha_2 \alpha_3^2 + 7 \beta_5 \alpha_2 \mu_4 \alpha_3^2 + 5 \beta_5 \alpha_3 \mu_1 \alpha_1^2 + 24 \mu_3 \beta_2 \alpha_1^2 \alpha_3 \\
& - \beta_5 \mu_4 \alpha_3 \alpha_1^2 + 24 \mu_3 \beta_3 \alpha_3^2 \alpha_1 - 16 \alpha_3^2 \beta_2 \alpha_1 \mu_1 - 24 \beta_6 \alpha_1^2 \alpha_3 \mu_3) x_1^2 x_3 / (
\end{aligned}$$

$$\begin{aligned}
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} (8 \beta_5^2 \alpha_3 \alpha_2 \alpha_1 + 16 \beta_5 \alpha_2 \beta_6 \alpha_3^2 + 8 \beta_5 \beta_3 \alpha_3^2 \alpha_1 \\
& + 8 \beta_5 \alpha_3 \beta_1 \alpha_2 \alpha_1 + 16 \beta_6 \beta_3 \alpha_3^3 + 32 \beta_6 \beta_4 \alpha_1^3 - 16 \beta_6^2 \alpha_3^2 \alpha_1 - 16 \beta_6 \alpha_3^2 \beta_1 \alpha_2 \\
& - 16 \beta_6 \alpha_3 \beta_1 \alpha_1^2 - 8 \beta_5 \beta_6 \alpha_1^2 \alpha_3 - 16 \alpha_3 \beta_1^2 \alpha_1 \alpha_2 + 16 \beta_3 \alpha_3^2 \alpha_1 \beta_1 - 16 \beta_2^2 \alpha_3^2 \alpha_1 \\
& - 8 \beta_1 \beta_4 \alpha_2^2 \alpha_3 + 4 \beta_1 \beta_2 \alpha_1^2 \alpha_3 + 4 \beta_1 \alpha_3^2 \beta_2 \alpha_2 - 32 \beta_2 \beta_4 \alpha_1^3 - 16 \beta_2 \beta_3 \alpha_3^3 \\
& + 56 \beta_1 \beta_4 \alpha_1^2 \alpha_2 - 4 \beta_5 \alpha_3 \alpha_2^2 \beta_4 - 24 \beta_3 \alpha_3 \alpha_1^2 \beta_4 - 4 \beta_2 \beta_5 \alpha_3^2 \alpha_2 + 20 \beta_2 \beta_5 \alpha_3 \alpha_1^2 \\
& - 44 \alpha_2 \beta_5 \alpha_1^2 \beta_4 + 24 \beta_3 \alpha_3^2 \alpha_2 \beta_4 + 48 \beta_4^2 \alpha_1 \alpha_2^2 + 32 \beta_2 \beta_6 \alpha_3^2 \alpha_1) x_1^2 x_2 / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \frac{1}{24} (24 \beta_5 \beta_6 \alpha_1^3 - 24 \alpha_2 \beta_5^2 \alpha_1^2 + 16 \beta_4^2 \alpha_2^3 \\
& - 16 \alpha_2 \beta_6 \alpha_3 \alpha_1 \beta_5 - 4 \alpha_3^2 \beta_2^2 \alpha_2 + 24 \beta_6^2 \alpha_1^2 \alpha_3 - 4 \beta_6 \alpha_3^2 \beta_2 \alpha_2 + 8 \beta_4 \alpha_2^2 \beta_5 \alpha_1 \\
& - 36 \beta_2 \beta_6 \alpha_1^2 \alpha_3 - 48 \beta_3 \beta_4 \alpha_2 \alpha_3 \alpha_1 + 8 \alpha_3^2 \beta_1 \alpha_2 \beta_3 + 16 \beta_2 \beta_5 \alpha_3 \alpha_2 \alpha_1 \\
& + 24 \beta_1 \alpha_2 \beta_5 \alpha_1^2 + 16 \beta_4 \beta_1 \alpha_2^2 \alpha_1 - 4 \beta_5 \beta_3 \alpha_3 \alpha_1^2 - 16 \beta_1 \alpha_3 \beta_2 \alpha_2 \alpha_1 - 16 \beta_3 \beta_4 \alpha_1^3 \\
& - 8 \alpha_3 \beta_1 \alpha_1^2 \beta_3 - 16 \beta_3^2 \alpha_3^3 + 12 \beta_2^2 \alpha_1^2 \alpha_3 + 16 \beta_1 \alpha_2 \beta_6 \alpha_3 \alpha_1 + 4 \beta_5 \beta_3 \alpha_3^2 \alpha_2 \\
& + 8 \beta_6^2 \alpha_3^2 \alpha_2 - 24 \beta_2 \beta_5 \alpha_1^3) x_1 x_2^2 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) - \frac{1}{24} ( \\
& 15 \beta_4 \mu_2 \alpha_3^2 \alpha_2 - 15 \beta_4 \alpha_2^2 \mu_3 \alpha_3 + 9 \beta_4 \alpha_2 \mu_3 \alpha_1^2 + 18 \mu_3 \beta_5 \alpha_3 \alpha_2 \alpha_1 - 18 \mu_3 \alpha_3 \beta_1 \alpha_2 \alpha_1 \\
& + 12 \beta_6 \alpha_3^2 \alpha_1 \mu_1 + 15 \beta_4 \mu_2 \alpha_3 \alpha_1^2 + 9 \mu_1 \alpha_2 \alpha_3^2 \beta_1 + 12 \mu_4 \beta_3 \alpha_3^3 + 12 \mu_4 \beta_4 \alpha_2 \alpha_3 \alpha_1 \\
& - 12 \mu_4 \beta_6 \alpha_3^2 \alpha_1 + 12 \mu_4 \alpha_3^2 \beta_2 \alpha_1 + 3 \mu_4 \alpha_3 \beta_1 \alpha_1^2 + 6 \mu_3 \beta_1 \alpha_1^3 - 6 \beta_5 \mu_3 \alpha_1^3 \\
& - 12 \beta_3 \alpha_3^3 \mu_1 - 3 \alpha_3 \mu_1 \alpha_1^2 \beta_1 - 9 \mu_4 \alpha_3^2 \beta_1 \alpha_2 - 12 \beta_4 \alpha_2 \alpha_3 \alpha_1 \mu_1 - 9 \beta_5 \mu_1 \alpha_2 \alpha_3^2 \\
& + 9 \beta_5 \alpha_2 \mu_4 \alpha_3^2 + 3 \beta_5 \alpha_3 \mu_1 \alpha_1^2 + 24 \mu_3 \beta_2 \alpha_1^2 \alpha_3 - 3 \beta_5 \mu_4 \alpha_3 \alpha_1^2 + 24 \mu_3 \beta_3 \alpha_3^2 \alpha_1 \\
& - 12 \alpha_3^2 \beta_2 \alpha_1 \mu_1 - 24 \beta_6 \alpha_1^2 \alpha_3 \mu_3) x_1^2 x_3 / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) + \gamma_8 x_2^3 \\
& + \gamma_5 x_1^3, 0 \Big]
\end{aligned}$$

```

> for j from 2 to targetspace do
>   print("Transformation from degree ", j-1, " into degree
", j, " is exp(ad(Trafo)) with Trafo=");
   print(Trafo[j]);
> od;

```

"Transformation from degree ", 1, " into degree ", 2, " is exp(ad(Trafo)) with Trafo="

$$\begin{aligned}
& \left[ \frac{1}{12} (-3 \alpha_2^2 \mu_4 x_3 \alpha_3 x_2 + 3 \mu_1 x_3 \alpha_2^2 \alpha_3 x_2 + 4 \beta_5 x_1 \alpha_3 \alpha_2^2 x_2 + 8 \beta_5 x_1^2 \alpha_3 \alpha_2 \alpha_1 \right. \\
& + 24 \beta_1 x_1 \alpha_1^2 \alpha_2 x_2 + 3 \mu_1 x_3 \alpha_1^2 \alpha_2 x_2 + 16 \beta_4 x_1 \alpha_1 \alpha_2^2 x_2 + 8 \beta_3 \alpha_3^2 x_1 \alpha_2 x_2 \\
& + 8 \alpha_3 \beta_1 x_1 \alpha_2^2 x_2 + 4 \alpha_3 \beta_1 x_1^2 \alpha_2 \alpha_1 + 3 \mu_2 x_3 \alpha_3 \alpha_1^2 x_1 - 3 \alpha_2 \mu_3 x_3 \alpha_1^2 x_1 \\
& \left. - 6 \mu_2 x_3 \alpha_3 \alpha_1 \alpha_2 x_2 - 8 \beta_3 x_2 \alpha_3 \alpha_1^2 x_1 - 12 \beta_3 x_2^2 \alpha_3 \alpha_1 \alpha_2 + 4 \alpha_3 \beta_2 \alpha_2 x_2 \alpha_1 x_1 \right]
\end{aligned}$$

$$\begin{aligned}
& -16 \alpha_2 \beta_6 x_2 \alpha_3 \alpha_1 x_1 - 4 \beta_3 x_2^2 \alpha_1^3 + 12 \beta_1 x_1^2 \alpha_1^3 - 12 \alpha_2 \beta_5 x_2 \alpha_1^2 x_1 + 8 \beta_4 \alpha_2^3 x_2^2 \\
& - 3 \alpha_2^2 \mu_3 x_3 \alpha_3 x_1 + 3 \mu_2 x_3 \alpha_3^2 \alpha_2 x_1 - 3 \alpha_2 \mu_4 x_3 \alpha_1^2 x_2 - 4 \beta_2 x_2^2 \alpha_1^2 \alpha_2 \\
& - 12 \beta_2 x_2 \alpha_1^3 x_1 + 4 \alpha_3 \beta_2 \alpha_2^2 x_2^2 - 4 \alpha_2^2 \beta_6 x_2^2 \alpha_3 + 8 \beta_3 \alpha_3^2 x_1^2 \alpha_1 + 4 \beta_4 x_1^2 \alpha_1^2 \alpha_2 \\
& - 6 \mu_2 x_3 \alpha_1^3 x_2 - 8 \alpha_2^2 \beta_5 x_2^2 \alpha_1 + 4 \alpha_3^2 \beta_2 x_1^2 \alpha_2 + 8 \alpha_2 \beta_6 \alpha_3^2 x_1^2 + 4 \alpha_2 \beta_6 x_2^2 \alpha_1^2 \\
& + 8 \beta_1 \alpha_2^2 x_2^2 \alpha_1 - 4 \beta_4 x_1^2 \alpha_2^2 \alpha_3 + 12 \beta_2 \alpha_1^2 x_1^2 \alpha_3) / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4), \\
& \frac{1}{12} (-12 \beta_6 x_2^2 \alpha_1^3 + 4 \beta_4 x_1^2 \alpha_1^3 + 8 \beta_3 \alpha_3^3 x_1^2 + 8 \alpha_3 \beta_1 \alpha_2^2 x_2^2 + 4 \beta_5 \alpha_3 \alpha_2^2 x_2^2 \\
& - 4 \beta_3 x_2^2 \alpha_3^2 \alpha_2 + 4 \alpha_3 \beta_1 x_1^2 \alpha_1^2 + 8 \alpha_3^2 \beta_2 x_1^2 \alpha_1 + 12 \beta_5 x_2^2 \alpha_1^2 \alpha_2 + 12 \beta_5 x_2 \alpha_1^3 x_1 \\
& - 8 \beta_6 \alpha_3^2 x_1^2 \alpha_1 + 6 \mu_3 x_3 \alpha_1^3 x_1 - 8 \beta_4 \alpha_1 \alpha_2^2 x_2^2 - 4 \beta_5 x_1^2 \alpha_3 \alpha_1^2 - 4 \alpha_3^2 \beta_1 x_1^2 \alpha_2 \\
& + 4 \beta_5 x_1^2 \alpha_3^2 \alpha_2 + 4 \beta_3 x_2^2 \alpha_3 \alpha_1^2 - 3 \mu_2 x_3 \alpha_3 \alpha_1^2 x_2 + 24 \beta_6 x_2 \alpha_1^2 \alpha_3 x_1 \\
& + 16 \alpha_3 \beta_1 x_1 \alpha_1 \alpha_2 x_2 - 4 \alpha_2 \beta_6 x_2^2 \alpha_3 \alpha_1 - 3 \mu_2 x_3 \alpha_3^2 \alpha_2 x_2 + 3 \mu_4 x_3 \alpha_3 \alpha_1^2 x_1 \\
& + 8 \beta_6 \alpha_3^2 x_1 \alpha_2 x_2 - 16 \beta_3 x_2 \alpha_3^2 \alpha_1 x_1 + 8 \beta_4 x_1 \alpha_2^2 \alpha_3 x_2 + 6 \alpha_2 \mu_3 x_3 \alpha_3 \alpha_1 x_1 \\
& + 12 \beta_4 x_1^2 \alpha_2 \alpha_3 \alpha_1 - 3 \alpha_3 \mu_1 x_3 \alpha_1^2 x_1 + 3 \alpha_2^2 \mu_3 x_3 \alpha_3 x_2 + 4 \alpha_3^2 \beta_2 x_1 \alpha_2 x_2 \\
& + 3 \mu_3 x_3 \alpha_1^2 \alpha_2 x_2 - 4 \beta_5 x_1 \alpha_3 \alpha_1 \alpha_2 x_2 - 8 \beta_4 \alpha_1^2 \alpha_2 x_2 x_1 - 12 \alpha_3 \beta_2 x_2 \alpha_1^2 x_1 \\
& + 3 \alpha_2 \mu_4 x_3 \alpha_3^2 x_1 - 8 \alpha_3 \beta_2 \alpha_2 x_2^2 \alpha_1 - 3 \mu_1 x_3 \alpha_2 \alpha_3^2 x_1) / ( \\
& \alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4), 0 \Big]
\end{aligned}$$

"Transformation from degree ", 2, " into degree ", 3, " is exp(ad(Trafo)) with Trafo="

$$\begin{aligned}
& \left[ \frac{1}{288} (72 x_2 x_3^2 \alpha_3 \alpha_1^3 \mu_2^2 + 72 x_2 x_3^2 \mu_1 \mu_2 \alpha_1^4 + 72 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_2 \beta_5 \alpha_1 \right. \\
& - 288 x_3 x_2^2 \alpha_2^2 \alpha_3^2 v_3 \alpha_1 + 180 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_3 \mu_3 \alpha_1 + 316 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_1 \beta_6 \alpha_1 \\
& - 316 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_1 \beta_2 \alpha_1 - 312 x_3 x_2^2 \alpha_2^4 \beta_4 \mu_3 + 192 x_3 x_2^2 \alpha_2^4 \alpha_3 v_4 \\
& + 36 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_2 \beta_6 - 184 x_3 x_2^2 \alpha_2^2 \beta_1 \mu_1 \alpha_1^2 - 4 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \beta_3 \mu_4 \\
& + 144 x_1 x_3^2 \alpha_2 \alpha_3^2 \rho_2 \alpha_1^2 + 36 x_1 x_3^2 \alpha_2^3 \alpha_3 \mu_3^2 - 36 x_1 x_3^2 \alpha_2^2 \alpha_3 \mu_4 \mu_3 \alpha_1 \\
& - 24 x_3 x_2^2 \beta_2 \mu_2 \alpha_1^4 + 24 x_3 x_2^2 \mu_2 \beta_6 \alpha_1^4 + 12 x_3 x_2^2 \alpha_3 \mu_2 \beta_3 \alpha_1^3 \\
& + 184 x_3 x_2^2 \alpha_2^2 \mu_1 \beta_5 \alpha_1^2 + 32 x_3 x_2^2 \mu_1 \beta_3 \alpha_1^4 - 64 x_3 x_2^2 \beta_3 \alpha_1^4 \mu_4 \\
& - 36 x_1 x_3^2 \alpha_2 \alpha_3^2 \mu_1 \mu_2 \alpha_1 + 36 x_1 x_3^2 \alpha_2^2 \alpha_3 \mu_1 \mu_3 \alpha_1 - 36 x_1 x_3^2 \alpha_3^2 \mu_2^2 \alpha_1^2 \\
& + 72 x_1 x_3^2 \alpha_3 \rho_2 \alpha_1^4 - 36 x_1 x_3^2 \alpha_3 \mu_1 \mu_2 \alpha_1^3 + 36 x_1 x_3^2 \alpha_2 \alpha_3^2 \mu_2 \mu_4 \alpha_1 \\
& - 72 x_1 x_3^2 \alpha_2^3 \alpha_3 \rho_3 + 36 x_1 x_3^2 \alpha_2 \mu_3 \alpha_1^3 \mu_1 + 36 x_1 x_3^2 \alpha_2^2 \alpha_1^2 \mu_3^2 \\
& \left. + 72 x_1 x_3^2 \alpha_2^2 \alpha_3^3 \rho_2 - 36 x_1 x_3^2 \alpha_2 \alpha_3^3 \mu_2^2 - 36 x_1 x_3^2 \alpha_2 \mu_4 \mu_3 \alpha_1^3 \right]
\end{aligned}$$

$$\begin{aligned}
& + 36 x_2 x_3^2 \alpha_2^2 \mu_4 \mu_3 \alpha_1^2 + 36 x_2 x_3^2 \alpha_2 \alpha_3 \alpha_1^2 \mu_1 \mu_2 - 36 x_2 x_3^2 \alpha_2^3 \alpha_3 \mu_1 \mu_3 \\
& - 72 x_2 x_3^2 \alpha_2 \rho_4 \alpha_1^4 - 36 x_2 x_3^2 \alpha_2 \alpha_3 \alpha_1^2 \mu_2 \mu_4 - 72 x_2 x_3^2 \alpha_2^3 \alpha_3^2 \rho_4 \\
& + 72 x_2 x_3^2 \alpha_2^3 \alpha_3^2 \rho_1 + 72 x_2 x_3^2 \alpha_2 \rho_1 \alpha_1^4 - 36 x_2 x_3^2 \alpha_2 \mu_1^2 \alpha_1^3 - 36 x_2 x_3^2 \alpha_2 \mu_4^2 \alpha_1^3 \\
& + 72 x_2 x_3^2 \alpha_2 \mu_1 \mu_4 \alpha_1^3 - 288 x_2 x_3^2 \alpha_2 \alpha_3 \rho_2 \alpha_1^3 + 72 x_2 x_3^2 \alpha_2 \alpha_1 \alpha_3^2 \mu_2^2 \\
& + 72 x_2 x_3^2 \alpha_2 \mu_2 \mu_3 \alpha_1^3 - 36 x_2 x_3^2 \alpha_2^2 \alpha_3 \alpha_1 \mu_4^2 - 144 x_1 x_3^2 \alpha_2^2 \alpha_3 \rho_3 \alpha_1^2 \\
& + 36 x_1 x_3^2 \alpha_3 \mu_2 \mu_4 \alpha_1^3 - 72 x_2 x_3^2 \mu_2 \alpha_1^4 \mu_4 + 72 x_2 x_3^2 \alpha_2^2 \alpha_3 \alpha_1 \mu_2 \mu_3 \\
& - 36 x_2 x_3^2 \alpha_2^2 \alpha_3 \alpha_1 \mu_1^2 + 72 x_2 x_3^2 \alpha_2^2 \alpha_3 \alpha_1 \mu_1 \mu_4 - 144 x_2 x_3^2 \alpha_2^2 \alpha_3 \rho_4 \alpha_1^2 \\
& + 144 x_2 x_3^2 \alpha_2^2 \alpha_3 \rho_1 \alpha_1^2 - 36 x_2 x_3^2 \alpha_2^2 \alpha_3^2 \mu_1 \mu_2 - 144 x_2 x_3^2 \alpha_2^2 \alpha_3^2 \rho_2 \alpha_1 \\
& + 36 x_2 x_3^2 \alpha_2^2 \alpha_3^2 \mu_2 \mu_4 - 36 x_2 x_3^2 \alpha_2^2 \mu_1 \mu_3 \alpha_1^2 + 36 x_2 x_3^2 \alpha_2^3 \alpha_3 \mu_4 \mu_3 \\
& - 72 x_1 x_3^2 \alpha_2 \rho_3 \alpha_1^4 + 432 x_3 x_2^2 \alpha_2^2 \mu_2 \beta_4 \alpha_1^2 - 72 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_1 \mu_2 \alpha_1 \\
& - 380 x_3 x_2^2 \alpha_2 \beta_3 \mu_4 \alpha_3 \alpha_1^2 - 312 x_3 x_2^2 \alpha_2^3 \mu_1 \beta_4 \alpha_1 - 252 x_3 x_2^2 \alpha_2^2 \beta_6 \alpha_1^2 \mu_3 \\
& + 432 x_1 x_2 x_3 \beta_1 \mu_2 \alpha_1^4 + 372 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \mu_4 \beta_5 \alpha_1 - 372 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \mu_1 \beta_5 \alpha_1 \\
& + 120 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \beta_1 \mu_1 \alpha_1 - 72 x_2^3 \gamma_4 \alpha_1^5 - 192 x_1 x_2 x_3 \alpha_2^2 \alpha_3 v_5 \alpha_1^2 \\
& + 504 x_1 x_2 x_3 \alpha_2 \beta_1 \mu_4 \alpha_1^3 + 120 x_3 x_1^2 \mu_2 \beta_4 \alpha_1^4 - 60 x_3 x_1^2 \alpha_2^2 \alpha_3 \beta_1 \mu_3 \alpha_1 \\
& - 268 x_3 x_1^2 \alpha_2^2 \alpha_3 \mu_4 \beta_4 \alpha_1 + 40 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \mu_1 \beta_5 + 96 x_3 x_1^2 \alpha_2^2 \alpha_3^2 v_1 \alpha_1 \\
& - 192 x_3 x_1^2 \alpha_2^2 \alpha_3 \beta_5 \mu_3 \alpha_1 - 148 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \beta_1 \mu_4 - 12 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \beta_2 \mu_3 \\
& - 24 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \beta_6 \mu_3 - 104 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \mu_4 \beta_5 + 96 x_3 x_1^2 \alpha_2^2 \alpha_3^3 v_2 \\
& + 140 x_3 x_1^2 \alpha_2^2 \alpha_3 \mu_1 \beta_4 \alpha_1 + 192 x_3 x_1^2 \alpha_2^2 \alpha_3^3 v_6 + 96 x_3 x_1^2 \alpha_2 v_4 \alpha_1^4 \\
& + 48 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \mu_2 \beta_4 \alpha_1 - 192 x_1 x_2 x_3 \alpha_3 v_3 \alpha_1^4 + 288 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \mu_3 \\
& - 120 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \beta_1 \mu_4 \alpha_1 + 88 x_1 x_2 x_3 \alpha_3 \mu_1 \beta_3 \alpha_1^3 + 492 x_1 x_2 x_3 \alpha_3 \mu_2 \beta_2 \alpha_1^3 \\
& - 240 x_1 x_2 x_3 \alpha_3 \mu_2 \beta_6 \alpha_1^3 + 408 x_1 x_2 x_3 \alpha_3^2 \mu_2 \beta_3 \alpha_1^2 - 288 x_1 x_2 x_3 v_2 \alpha_1^5 \\
& - 668 x_1 x_2 x_3 \alpha_2 \mu_1 \beta_2 \alpha_1^2 \alpha_3 - 208 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \mu_1 \beta_6 + 96 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 v_2 \alpha_1 \\
& + 80 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_6 \mu_4 - 12 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \mu_2 \beta_5 - 24 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_1 \mu_2 \\
& - 116 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_2 \mu_4 - 384 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 v_6 \alpha_1 + 540 x_1 x_2 x_3 \alpha_2^2 \beta_5 \mu_3 \alpha_1^2 \\
& + 32 x_1 x_2 x_3 \alpha_2^3 \alpha_3 \mu_1 \beta_4 - 168 x_1 x_2 x_3 \alpha_2^3 \alpha_3 \beta_1 \mu_3 - 84 x_1 x_2 x_3 \alpha_2^3 \alpha_3 \beta_5 \mu_3 \\
& + 768 x_1 x_2 x_3 \alpha_2^2 \alpha_3 v_1 \alpha_1^2 + 192 x_1 x_2 x_3 \alpha_2^2 \alpha_3^3 v_3 + 220 x_1 x_2 x_3 \alpha_2 \beta_2 \mu_4 \alpha_3 \alpha_1^2 \\
& - 288 x_1 x_2 x_3 \beta_2 \alpha_1^4 \mu_4 - 240 x_1 x_2 x_3 \beta_3 \mu_3 \alpha_1^4 - 216 x_1 x_2 x_3 \mu_2 \beta_5 \alpha_1^4 \\
& + 408 x_1 x_2 x_3 \alpha_2 \beta_3 \alpha_3 \alpha_1^2 \mu_3 + 396 x_1 x_2 x_3 \alpha_2 \mu_2 \beta_5 \alpha_3 \alpha_1^2 - 64 x_3 x_2^2 \alpha_2^3 \alpha_3 \beta_1 \mu_4
\end{aligned}$$

$$\begin{aligned}
& + 128 x_3 x_2^2 \alpha_2^3 \alpha_3 \beta_1 \mu_1 + 312 x_3 x_2^2 \alpha_2^3 \beta_5 \alpha_1 \mu_3 + 312 x_3 x_2^2 \alpha_2^3 \beta_4 \alpha_1 \mu_4 \\
& - 312 x_3 x_2^2 \alpha_2^3 \beta_1 \alpha_1 \mu_3 - 192 x_3 x_2^2 \alpha_2^3 \alpha_3 v_5 \alpha_1 + 192 x_3 x_2^2 \alpha_2^3 \alpha_3 v_1 \alpha_1 \\
& + 120 x_3 x_2^2 \alpha_2^3 \alpha_3 \mu_2 \beta_4 - 248 x_3 x_2^2 \alpha_2^2 \mu_4 \alpha_1^2 \beta_5 + 248 x_3 x_2^2 \alpha_2^2 \beta_1 \mu_4 \alpha_1^2 \\
& + 324 x_3 x_2^2 \alpha_2 \mu_2 \beta_3 \alpha_3^2 \alpha_1 - 252 x_3 x_2^2 \alpha_2 \mu_2 \beta_6 \alpha_1^2 \alpha_3 - 96 x_3 x_2^2 \alpha_2^3 \alpha_3^2 v_6 \\
& - 4 x_3 x_2^2 \alpha_2 \beta_2 \alpha_1^3 \mu_1 - 384 x_3 x_2^2 \alpha_2 \alpha_3 v_3 \alpha_1^3 + 4 x_3 x_2^2 \alpha_2 \beta_6 \mu_1 \alpha_1^3 \\
& - 124 x_3 x_2^2 \alpha_2 \mu_4 \beta_2 \alpha_1^3 + 240 x_3 x_2^2 \alpha_2 \beta_1 \mu_2 \alpha_1^3 - 132 x_3 x_2^2 \alpha_2 \beta_3 \mu_3 \alpha_1^3 \\
& - 240 x_3 x_2^2 \alpha_2 \mu_2 \beta_5 \alpha_1^3 + 124 x_3 x_2^2 \alpha_2 \mu_4 \beta_6 \alpha_1^3 - 288 x_3 x_1^2 \mu_1 \beta_1 \alpha_1^4 \\
& - 192 x_3 x_2^2 \alpha_2^2 v_5 \alpha_1^3 + 192 x_3 x_2^2 \alpha_2^2 v_1 \alpha_1^3 - 220 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \mu_1 \beta_3 \\
& + 252 x_3 x_2^2 \alpha_2^2 \beta_2 \mu_3 \alpha_1^2 - 36 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \beta_2 \mu_2 - 128 x_3 x_2^2 \alpha_2^3 \alpha_3 \mu_1 \beta_5 \\
& + 188 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_2 \mu_4 \alpha_1 - 188 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \mu_4 + 60 x_3 x_2^2 \alpha_2^3 \alpha_3 \beta_6 \mu_3 \\
& + 64 x_3 x_2^2 \alpha_2^3 \alpha_3 \mu_4 \beta_5 - 96 x_3 x_2^2 \alpha_2 v_2 \alpha_1^4 + 96 x_3 x_2^2 \alpha_2 v_6 \alpha_1^4 \\
& - 60 x_3 x_2^2 \alpha_2^3 \alpha_3 \beta_2 \mu_3 + 96 x_3 x_2^2 \alpha_2^3 \alpha_3^2 v_2 + 124 x_3 x_2^2 \alpha_2 \mu_1 \beta_3 \alpha_3 \alpha_1^2 \\
& + 252 x_3 x_2^2 \alpha_2 \beta_2 \mu_2 \alpha_3 \alpha_1^2 + 192 x_3 x_2^2 \alpha_2 v_4 \alpha_1^2 + 39 x_2^3 \alpha_2^3 \alpha_3 \beta_1 \beta_2 \\
& - 24 x_2^3 \alpha_2^3 \alpha_3 \beta_2 \beta_5 + 96 x_2^3 \alpha_2^3 \alpha_3 \beta_3 \beta_4 - 36 x_2^3 \alpha_2 \beta_2 \beta_6 \alpha_1^3 + 72 x_2^3 \alpha_2 \beta_3^2 \alpha_3^2 \alpha_1 \\
& + 90 x_2^3 \alpha_2^3 \beta_4 \alpha_1 \beta_6 - 30 x_2^3 \alpha_2^2 \alpha_3 \beta_3 \alpha_1 \beta_1 - 75 x_2^3 \alpha_2^3 \beta_2 \beta_4 \alpha_1 \\
& + 144 x_2^3 \alpha_2 \beta_2 \beta_3 \alpha_3 \alpha_1^2 - 144 x_2^3 \alpha_2 \beta_3 \beta_6 \alpha_1^2 \alpha_3 + 228 x_2^2 x_1 \alpha_3 \beta_2 \beta_3 \alpha_1^3 \\
& - 63 x_2^2 x_1 \alpha_2^2 \alpha_3 \gamma_1 \alpha_1^2 + 9 x_2^3 \alpha_2^2 \alpha_3 \gamma_3 \alpha_1^2 - 9 x_2^3 \alpha_2^2 \alpha_3 \gamma_8 \alpha_1^2 \\
& - 102 x_2^3 \alpha_2^2 \beta_6 \alpha_1^2 \beta_5 + 72 x_2^3 \alpha_2^2 \beta_1 \beta_6 \alpha_1^2 - 66 x_2^3 \alpha_2^2 \beta_1 \beta_2 \alpha_1^2 \\
& + 96 x_2^3 \alpha_2^2 \beta_2 \beta_5 \alpha_1^2 - 63 x_2^3 \alpha_2^2 \alpha_3 \beta_2^2 \alpha_1 - 180 x_2^3 \alpha_2^2 \alpha_3^2 \gamma_4 \alpha_1 \\
& + 216 x_2^3 \alpha_2^2 \beta_3 \beta_4 \alpha_1^2 - 180 x_2^3 \alpha_2 \beta_3 \beta_5 \alpha_1^3 - 252 x_2^3 \alpha_2 \alpha_3 \gamma_4 \alpha_1^3 \\
& + 120 x_2^3 \alpha_2 \beta_3 \alpha_1^3 \beta_1 - 75 x_2^3 \alpha_2^3 \beta_1 \beta_5 \alpha_1 - 54 x_2^3 \alpha_2^3 \alpha_3 \gamma_7 \alpha_1 + 54 x_2^3 \alpha_2^3 \alpha_3 \gamma_2 \alpha_1 \\
& + 33 x_2^3 \alpha_2^3 \alpha_3 \beta_6 \beta_5 + 44 x_3 x_1^2 \alpha_2 \mu_4 \beta_4 \alpha_1^3 - 16 x_3 x_1^2 \alpha_2 \mu_4 \beta_3 \alpha_3^3 \\
& - 372 x_3 x_1^2 \alpha_2 \mu_3 \beta_1 \alpha_1^3 + 120 x_3 x_1^2 \alpha_2 \beta_5 \mu_3 \alpha_1^3 + 129 x_2^3 \alpha_2^2 \alpha_3 \beta_2 \beta_6 \alpha_1 \\
& + 18 x_2^3 \alpha_2^2 \alpha_3^2 \beta_3 \beta_6 - 27 x_2^3 \alpha_2^2 \alpha_3^2 \beta_2 \beta_3 - 15 x_2^3 \alpha_2^2 \alpha_3 \beta_5 \beta_3 \alpha_1 \\
& - 172 x_3 x_1^2 \alpha_2 \beta_4 \alpha_1^3 \mu_1 - 66 x_2^3 \alpha_2^2 \alpha_3 \beta_6^2 \alpha_1 - 48 x_2^3 \alpha_2^3 \alpha_3 \beta_1 \beta_6 \\
& + 48 x_2^2 x_1 \alpha_2^2 \beta_6 \alpha_1^2 \beta_4 + 39 x_2^2 x_1 \alpha_2^2 \alpha_3 \beta_1 \beta_2 \alpha_1 + 108 x_2^2 x_1 \alpha_2^2 \alpha_3 \beta_3 \beta_4 \alpha_1 \\
& - 6 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \beta_3 \beta_1 + 129 x_2^2 x_1 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \beta_5 - 84 x_2^2 x_1 \alpha_2^2 \alpha_3 \beta_1 \beta_6 \alpha_1 \\
& + 24 x_2^2 x_1 \alpha_2^2 \beta_2 \beta_4 \alpha_1^2 + 48 x_2^2 x_1 \alpha_2^2 \beta_5^2 \alpha_1^2 - 96 x_2^2 x_1 \alpha_2^2 \beta_1 \beta_5 \alpha_1^2
\end{aligned}$$

$$\begin{aligned}
& + 48 x_2^2 x_1 \alpha_2^2 \beta_1^2 \alpha_1^2 + 48 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \beta_6^2 + 21 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \beta_2^2 \\
& + 135 x_2^2 x_1 \alpha_2^2 \alpha_3^3 \gamma_4 - 72 x_2^2 x_1 \alpha_2 \gamma_7 \alpha_1^4 + 30 x_1^3 \beta_3^2 \alpha_3^4 - 12 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \mu_2 \beta_4 \\
& + 192 x_3 x_1^2 \alpha_2^2 \alpha_3^2 v_5 \alpha_1 - 204 x_3 x_1^2 \alpha_2^2 \beta_4 \alpha_1^2 \mu_3 - 204 x_3 x_1^2 \alpha_2 \beta_4 \mu_2 \alpha_3 \alpha_1^2 \\
& - 312 x_3 x_1^2 \alpha_2 \beta_6 \alpha_3^2 \alpha_1 \mu_1 - 460 x_3 x_1^2 \alpha_2 \mu_4 \alpha_3 \beta_1 \alpha_1^2 + 312 x_3 x_1^2 \alpha_2 \mu_4 \beta_6 \alpha_3^2 \alpha_1 \\
& - 60 x_3 x_1^2 \alpha_2 \mu_4 \alpha_3^2 \beta_2 \alpha_1 + 108 x_3 x_1^2 \alpha_2^3 \alpha_3 \beta_4 \mu_3 - 108 x_3 x_1^2 \alpha_2 \mu_3 \beta_2 \alpha_1^2 \alpha_3 \\
& + 208 x_3 x_1^2 \alpha_2 \beta_5 \mu_4 \alpha_3 \alpha_1^2 - 96 x_3 x_1^2 \alpha_2^3 \alpha_3^2 v_4 - 76 x_3 x_1^2 \alpha_2^2 \alpha_3^2 \beta_1 \mu_1 \\
& + 24 x_3 x_1^2 \alpha_2 \mu_3 \beta_3 \alpha_3^2 \alpha_1 + 60 x_3 x_1^2 \alpha_2 \alpha_3^2 \beta_2 \alpha_1 \mu_1 + 288 x_3 x_1^2 \alpha_2 \beta_6 \alpha_1^2 \alpha_3 \mu_3 \\
& - 144 x_3 x_1^2 \alpha_2 \alpha_3^2 \mu_2 \beta_5 \alpha_1 - 108 x_3 x_1^2 \alpha_2 \alpha_3^2 \beta_1 \mu_2 \alpha_1 - 272 x_3 x_1^2 \alpha_2 \beta_5 \alpha_3 \mu_1 \alpha_1^2 \\
& - 52 x_3 x_1^2 \alpha_2 \alpha_3 \mu_1 \alpha_1^2 \beta_1 + 288 x_3 x_1^2 \alpha_3 v_2 \alpha_1^4 - 168 x_3 x_1^2 \alpha_2 \alpha_3^3 \mu_2 \beta_6 \\
& + 192 x_3 x_1^2 \alpha_2 \alpha_3 v_5 \alpha_1^3 + 384 x_3 x_1^2 \alpha_2 \alpha_3 v_1 \alpha_1^3 + 192 x_3 x_1^2 \alpha_2 \alpha_3^2 v_6 \alpha_1^2 \\
& + 384 x_3 x_1^2 \alpha_2 \alpha_3^2 v_2 \alpha_1^2 + 192 x_3 x_1^2 \alpha_2 \alpha_3^3 v_3 \alpha_1 - 84 x_3 x_1^2 \alpha_2 \alpha_3^3 \beta_2 \mu_2 \\
& + 80 x_3 x_1^2 \alpha_2 \beta_3 \alpha_3^3 \mu_1 - 252 x_3 x_1^2 \alpha_3 \beta_2 \alpha_1^3 \mu_1 - 312 x_3 x_1^2 \alpha_3^3 \mu_2 \beta_3 \alpha_1 \\
& - 232 x_3 x_1^2 \alpha_3^2 \beta_3 \mu_1 \alpha_1^2 + 296 x_3 x_1^2 \alpha_3^2 \mu_4 \beta_3 \alpha_1^2 - 396 x_3 x_1^2 \alpha_3^2 \beta_2 \mu_2 \alpha_1^2 \\
& + 144 x_3 x_1^2 \alpha_3^2 \mu_2 \beta_6 \alpha_1^2 - 420 x_3 x_1^2 \alpha_3 \beta_1 \mu_2 \alpha_1^3 + 168 x_3 x_1^2 \alpha_3 \mu_2 \beta_5 \alpha_1^3 \\
& + 252 x_3 x_1^2 \alpha_3 \mu_4 \beta_2 \alpha_1^3 + 336 x_3 x_1^2 \alpha_3 \mu_3 \beta_3 \alpha_1^3 + 216 x_3 x_1^2 \beta_2 \mu_3 \alpha_1^4 \\
& - 544 x_1 x_2 x_3 \alpha_2 \beta_6 \mu_4 \alpha_3 \alpha_1^2 - 216 x_1 x_2 x_3 \alpha_2 \beta_1 \mu_2 \alpha_3 \alpha_1^2 \\
& + 280 x_1 x_2 x_3 \alpha_2 \alpha_3^2 \mu_4 \beta_3 \alpha_1 + 416 x_1 x_2 x_3 \alpha_2 \beta_6 \alpha_3 \mu_1 \alpha_1^2 \\
& - 132 x_1 x_2 x_3 \alpha_2 \mu_2 \alpha_3^2 \beta_2 \alpha_1 + 384 x_1 x_2 x_3 \alpha_2 \mu_2 \beta_6 \alpha_3^2 \alpha_1 \\
& - 536 x_1 x_2 x_3 \alpha_2 \mu_1 \beta_3 \alpha_3^2 \alpha_1 - 344 x_1 x_2 x_3 \alpha_3 \mu_4 \beta_3 \alpha_1^3 + 464 x_1 x_2 x_3 \alpha_2^2 \mu_4 \beta_4 \alpha_1^2 \\
& - 592 x_1 x_2 x_3 \alpha_2^2 \mu_1 \beta_4 \alpha_1^2 + 384 x_1 x_2 x_3 \alpha_2^2 v_4 \alpha_1^3 + 24 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_3 \mu_3 \\
& - 44 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \mu_1 \beta_2 - 192 x_1 x_2 x_3 \alpha_2 \alpha_3 v_2 \alpha_1^3 - 384 x_1 x_2 x_3 \alpha_2 \alpha_3 v_6 \alpha_1^3 \\
& - 252 x_1 x_2 x_3 \alpha_2 \mu_4 \alpha_1^3 \beta_5 - 216 x_1 x_2 x_3 \alpha_2 \mu_2 \beta_3 \alpha_3^3 + 588 x_1 x_2 x_3 \alpha_2 \beta_2 \mu_3 \alpha_1^3 \\
& - 336 x_1 x_2 x_3 \alpha_2 \beta_6 \mu_3 \alpha_1^3 + 672 x_1 x_2 x_3 \alpha_2 \mu_2 \beta_4 \alpha_1^3 + 252 x_1 x_2 x_3 \alpha_2 \beta_5 \alpha_1^3 \mu_1 \\
& + 288 x_3 x_1^2 v_1 \alpha_1^5 + 3 x_2^2 x_1 \alpha_2^3 \alpha_3 \beta_5^2 + 9 x_2^2 x_1 \alpha_2^3 \alpha_3^2 \gamma_7 - 126 x_2^2 x_1 \alpha_2 \alpha_3 \gamma_3 \alpha_1^3 \\
& + 27 x_2^2 x_1 \alpha_2 \alpha_3^2 \gamma_4 \alpha_1^2 - 162 x_2^2 x_1 \alpha_2 \alpha_3 \gamma_8 \alpha_1^3 + 96 x_2^2 x_1 \alpha_2 \beta_1 \beta_6 \alpha_1^3 \\
& + 288 x_2^2 x_1 \alpha_2 \beta_3 \beta_4 \alpha_1^3 - 96 x_2^2 x_1 \alpha_2 \beta_5 \beta_6 \alpha_1^3 + 96 x_2^2 x_1 \alpha_2 \beta_2 \beta_5 \alpha_1^3 \\
& - 114 x_2^2 x_1 \alpha_2 \beta_2^2 \alpha_1^2 \alpha_3 - 42 x_2^2 x_1 \alpha_2^3 \alpha_3 \beta_6 \beta_4 - 51 x_2^2 x_1 \alpha_2^3 \alpha_3 \beta_1 \beta_5 \\
& + 174 x_2^2 x_1 \alpha_2 \beta_5 \beta_3 \alpha_3 \alpha_1^2 - 84 x_2^2 x_1 \alpha_2 \alpha_3 \beta_1 \alpha_1^2 \beta_3 + 246 x_2^2 x_1 \alpha_2 \beta_2 \beta_6 \alpha_1^2 \alpha_3
\end{aligned}$$

$$\begin{aligned}
& + 186 x_2^2 x_1 \alpha_2 \alpha_3^2 \beta_3 \beta_6 \alpha_1 - 177 x_2^2 x_1 \alpha_2 \alpha_3^2 \beta_2 \beta_3 \alpha_1 + 81 x_2^2 x_1 \alpha_2^3 \gamma_5 \alpha_1^2 \\
& + 48 x_2^2 x_1 \alpha_2^3 \alpha_3 \beta_1^2 + 63 x_2^2 x_1 \alpha_2^3 \alpha_3^2 \gamma_2 + 144 x_2^2 x_1 \alpha_2 \gamma_2 \alpha_1^4 \\
& - 96 x_2^2 x_1 \alpha_2 \beta_1 \beta_2 \alpha_1^3 - 9 x_1^3 \alpha_2 \alpha_3^3 \beta_2^2 - 54 x_2^2 x_1 \alpha_2 \beta_3^2 \alpha_3^3 \\
& - 84 x_2^2 x_1 \alpha_2^2 \alpha_3 \beta_2 \beta_5 \alpha_1 - 132 x_2^2 x_1 \alpha_2 \beta_6^2 \alpha_1^2 \alpha_3 - 192 x_2 x_1^2 \alpha_3 \beta_3 \alpha_1^3 \beta_1 \\
& + 186 x_2 x_1^2 \alpha_2^2 \beta_5 \beta_4 \alpha_1^2 + 105 x_2 x_1^2 \alpha_2^2 \alpha_3 \beta_2 \beta_4 \alpha_1 - 48 x_2 x_1^2 \alpha_2^2 \alpha_3^2 \beta_1 \beta_6 \\
& + 21 x_2 x_1^2 \alpha_2^2 \alpha_3^2 \beta_1 \beta_2 + 24 x_2 x_1^2 \alpha_2^2 \alpha_3^2 \beta_2 \beta_5 + 81 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \gamma_8 \\
& + 24 x_2 x_1^2 \alpha_2 \beta_2 \beta_5 \alpha_3 \alpha_1^2 - 39 x_2 x_1^2 \alpha_2^3 \alpha_3 \beta_5 \beta_4 - 276 x_2 x_1^2 \alpha_2^2 \beta_1 \beta_4 \alpha_1^2 \\
& + 162 x_2 x_1^2 \alpha_2^2 \gamma_5 \alpha_1^3 - 72 x_2 x_1^2 \alpha_2 \gamma_6 \alpha_1^4 - 135 x_2 x_1^2 \alpha_2 \beta_2 \beta_6 \alpha_3^2 \alpha_1 \\
& + 57 x_2 x_1^2 \alpha_2 \beta_5 \beta_3 \alpha_3^2 \alpha_1 - 48 x_2 x_1^2 \alpha_2 \beta_6 \alpha_3 \beta_1 \alpha_1^2 + 48 x_2 x_1^2 \alpha_2 \beta_5 \beta_6 \alpha_1^2 \alpha_3 \\
& + 216 x_2 x_1^2 \alpha_2 \gamma_1 \alpha_1^4 - 102 x_2 x_1^2 \alpha_2 \beta_3 \alpha_3^2 \alpha_1 \beta_1 - 24 x_2 x_1^2 \alpha_2 \beta_1 \beta_2 \alpha_1^2 \alpha_3 \\
& + 135 x_2 x_1^2 \alpha_2^3 \alpha_3^2 \gamma_1 + 9 x_2 x_1^2 \alpha_2^3 \alpha_3^2 \gamma_6 - 63 x_2 x_1^2 \alpha_2^2 \alpha_3 \gamma_6 \alpha_1^2 \\
& - 180 x_2 x_1^2 \alpha_2^3 \beta_4^2 \alpha_1 + 162 x_2 x_1^2 \alpha_2 \alpha_3^3 \gamma_4 \alpha_1 + 72 x_1^3 \alpha_3 \gamma_2 \alpha_1^4 \\
& - 264 x_2^2 x_1 \alpha_3 \beta_3 \beta_6 \alpha_1^3 - 108 x_2^2 x_1 \alpha_3 \gamma_4 \alpha_1^4 + 216 x_2^2 x_1 \alpha_3^2 \beta_3^2 \alpha_1^2 \\
& + 192 x_2^2 x_1 \beta_1 \beta_3 \alpha_1^4 - 192 x_2^2 x_1 \beta_3 \beta_5 \alpha_1^4 + 81 x_2^2 x_1 \alpha_2^4 \alpha_3 \gamma_5 \\
& + 207 x_2^2 x_1 \alpha_2^2 \alpha_3 \gamma_2 \alpha_1^2 + 51 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \beta_5 \beta_3 - 162 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \gamma_8 \alpha_1 \\
& - 69 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \beta_2 \beta_6 + 18 x_2^2 x_1 \alpha_2^2 \alpha_3^2 \gamma_3 \alpha_1 + 144 x_1^3 \gamma_1 \alpha_1^5 \\
& + 117 x_1^3 \alpha_2 \alpha_3^2 \gamma_2 \alpha_1^2 + 27 x_1^3 \alpha_2 \alpha_3^2 \gamma_7 \alpha_1^2 + 9 x_1^3 \alpha_2 \alpha_3^3 \beta_2 \beta_6 \\
& - 90 x_1^3 \alpha_2 \beta_5 \alpha_3 \beta_1 \alpha_1^2 + 108 x_1^3 \alpha_2 \beta_6 \alpha_1^2 \alpha_3 \beta_4 + 45 x_1^3 \alpha_2 \beta_5 \beta_6 \alpha_3^2 \alpha_1 \\
& - 12 x_1^3 \alpha_2 \beta_5 \alpha_3^2 \beta_2 \alpha_1 + 36 x_1^3 \alpha_2 \beta_4 \beta_3 \alpha_3^2 \alpha_1 - 60 x_1^3 \alpha_2 \beta_6 \alpha_3^2 \alpha_1 \beta_1 \\
& - 18 x_1^3 \alpha_2 \beta_4 \beta_2 \alpha_1^2 \alpha_3 + 27 x_1^3 \alpha_2 \alpha_3^2 \beta_2 \alpha_1 \beta_1 + 24 x_1^3 \alpha_3^2 \beta_2 \beta_6 \alpha_1^2 \\
& + 18 x_1^3 \alpha_2^2 \alpha_3^2 \beta_6 \beta_4 - 75 x_1^3 \alpha_2^2 \alpha_3 \beta_5 \beta_4 \alpha_1 + 30 x_1^3 \alpha_2^2 \alpha_3 \beta_1 \beta_4 \alpha_1 \\
& - 9 x_1^3 \alpha_2^2 \alpha_3 \gamma_5 \alpha_1^2 - 9 x_1^3 \alpha_2^2 \alpha_3^2 \beta_2 \beta_4 + 54 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_6 \alpha_1 + 15 x_1^3 \alpha_2^2 \alpha_3^2 \beta_1 \beta_5 \\
& + 30 x_1^3 \alpha_2 \beta_5^2 \alpha_3 \alpha_1^2 + 60 x_1^3 \alpha_2 \alpha_3 \beta_1^2 \alpha_1^2 + 90 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_1 \alpha_1 \\
& + 234 x_1^3 \alpha_2 \alpha_3 \gamma_1 \alpha_1^3 + 54 x_1^3 \alpha_2 \alpha_3 \gamma_6 \alpha_1^3 + 30 x_1^3 \alpha_2 \beta_3 \alpha_3^3 \beta_1 - 120 x_1^3 \alpha_2 \beta_4 \beta_1 \alpha_1^3 \\
& - 15 x_1^3 \alpha_2 \beta_5 \beta_3 \alpha_3^3 + 207 x_2 x_1^2 \alpha_2 \alpha_3^2 \gamma_3 \alpha_1^2 + 240 x_2 x_1^2 \alpha_2 \beta_2 \beta_4 \alpha_1^3 \\
& + 15 x_2 x_1^2 \alpha_2 \beta_2 \beta_3 \alpha_3^3 + 45 x_2 x_1^2 \alpha_2 \beta_2^2 \alpha_3^2 \alpha_1 - 42 x_2 x_1^2 \alpha_2 \beta_6 \beta_3 \alpha_3^3 \\
& + 90 x_2 x_1^2 \alpha_2 \beta_6^2 \alpha_3^2 \alpha_1 - 96 x_2 x_1^2 \alpha_2 \beta_6 \beta_4 \alpha_1^3 + 162 x_2 x_1^2 \alpha_2^3 \alpha_3 \gamma_5 \alpha_1 \\
& - 6 x_2 x_1^2 \alpha_2^3 \alpha_3 \beta_1 \beta_4 + 90 x_2 x_1^2 \alpha_2^2 \alpha_3 \beta_1^2 \alpha_1 + 96 x_1^3 \alpha_3 \beta_4 \beta_3 \alpha_1^3
\end{aligned}$$



$$\begin{aligned}
& + 48 x_1^3 \alpha_3 \beta_5 \beta_2 \alpha_1^3 - 48 x_1^3 \alpha_3 \beta_2 \alpha_1^3 \beta_1 + 15 x_1^3 \alpha_3^3 \beta_2 \beta_3 \alpha_1 + 81 x_2 x_1^2 \alpha_2 \alpha_3^2 \gamma_8 \alpha_1^2 \\
& - 30 x_1^3 \alpha_3^3 \beta_6 \beta_3 \alpha_1 + 60 x_1^3 \alpha_2 \beta_5 \beta_4 \alpha_1^3 - 210 x_2 x_1^2 \alpha_3^2 \beta_2 \beta_3 \alpha_1^2 \\
& + 228 x_2 x_1^2 \alpha_3^2 \beta_6 \beta_3 \alpha_1^2 + 192 x_2 x_1^2 \alpha_3 \beta_5 \beta_3 \alpha_1^3 - 180 x_2 x_1^2 \alpha_3^3 \beta_3^2 \alpha_1 \\
& + 162 x_2 x_1^2 \alpha_3^2 \gamma_4 \alpha_1^3 - 6 x_2 x_1^2 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \beta_4 + 45 x_2 x_1^2 \alpha_2^2 \alpha_3 \beta_5^2 \alpha_1 \\
& + 351 x_2 x_1^2 \alpha_2^2 \alpha_3 \gamma_1 \alpha_1^2 + 63 x_2 x_1^2 \alpha_2^2 \alpha_3^3 \gamma_3 - 135 x_2 x_1^2 \alpha_2^2 \alpha_3 \beta_1 \beta_5 \alpha_1 \\
& + 3 x_2 x_1^2 \alpha_2^2 \alpha_3^2 \beta_6 \beta_5 + 45 x_2^2 x_1 \alpha_2^3 \beta_4 \beta_5 \alpha_1 + 69 x_2^2 x_1 \alpha_2^3 \alpha_3 \beta_2 \beta_4 \\
& - 90 x_2^2 x_1 \alpha_2^3 \beta_1 \beta_4 \alpha_1 + 144 x_2 x_1^2 \alpha_3 \gamma_3 \alpha_1^4 - 96 x_3 x_2^2 v_3 \alpha_1^5 - 144 x_2 x_3^2 \rho_2 \alpha_1^5 \\
& + 45 x_2^3 \alpha_2^3 \beta_5^2 \alpha_1 - 45 x_2^3 \alpha_2^4 \beta_4 \beta_5 + 27 x_2^3 \alpha_2^3 \gamma_6 \alpha_1^2 - 27 x_2^3 \alpha_2^4 \alpha_3 \gamma_1 \\
& + 36 x_2^3 \alpha_2 \gamma_8 \alpha_1^4 + 30 x_2^3 \alpha_2^4 \beta_1 \beta_4 - 24 x_2^3 \beta_2 \beta_3 \alpha_1^4 - 48 x_2^3 \alpha_3 \beta_3^2 \alpha_1^3 \\
& - 72 x_1^3 \alpha_2^2 \beta_4^2 \alpha_1^2 - 45 x_1^3 \alpha_2^3 \alpha_3^2 \gamma_5 - 27 x_1^3 \alpha_2 \gamma_4 \alpha_3^4 + 18 x_1^3 \alpha_2^3 \alpha_3 \beta_4^2 \\
& + 36 x_1^3 \alpha_2 \gamma_5 \alpha_1^4 - 90 x_2^2 x_1 \alpha_2^4 \beta_4^2 - 144 x_2^2 x_1 \gamma_3 \alpha_1^5 + 48 x_2^3 \beta_3 \beta_6 \alpha_1^4 \\
& - 54 x_2^3 \alpha_2^2 \gamma_7 \alpha_1^3 + 54 x_2^3 \alpha_2^2 \gamma_2 \alpha_1^3 + 27 x_2^3 \alpha_2^4 \alpha_3 \gamma_6 + 24 x_2^3 \alpha_2 \beta_6^2 \alpha_1^3 \\
& + 45 x_2^3 \alpha_2^3 \alpha_3^2 \gamma_3 - 27 x_2^3 \alpha_2^3 \gamma_1 \alpha_1^2 + 12 x_2^3 \alpha_2 \beta_2^2 \alpha_1^3 - 45 x_2^3 \alpha_2^3 \alpha_3^3 \gamma_8 \\
& - 36 x_2^3 \alpha_2 \gamma_3 \alpha_1^4 + 30 x_2^3 \alpha_2^3 \beta_1^2 \alpha_1 - 24 x_1^3 \alpha_3^2 \beta_2^2 \alpha_1^2 + 96 x_1^3 \beta_2 \beta_4 \alpha_1^4 \\
& - 27 x_1^3 \alpha_3^3 \gamma_4 \alpha_1^2 + 27 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_7 + 45 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_2 - 15 x_1^3 \alpha_2^2 \alpha_3^2 \beta_5^2 \\
& - 36 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \beta_2 \mu_3 \alpha_1 + 192 x_3 x_1^2 \alpha_3^2 v_3 \alpha_1^3 - 504 x_1 x_2 x_3 \alpha_2 \beta_1 \mu_1 \alpha_1^3 \\
& - 792 x_1 x_2 x_3 \alpha_2^2 \beta_1 \alpha_1^2 \mu_3 + 384 x_1 x_2 x_3 \alpha_2^3 \alpha_3 v_4 \alpha_1 - 160 x_1 x_2 x_3 \alpha_2^3 \alpha_3 \mu_4 \beta_4 \\
& - 624 x_1 x_2 x_3 \alpha_2^3 \beta_4 \alpha_1 \mu_3 + 576 x_1 x_2 x_3 \alpha_2 v_1 \alpha_1^4 - 288 x_1 x_2 x_3 \alpha_2 v_5 \alpha_1^4 \\
& + 96 x_1 x_2 x_3 \alpha_2^3 \alpha_3^2 v_5 + 192 x_1 x_2 x_3 \alpha_2^3 \alpha_3^2 v_1) / ((\alpha_2 \alpha_3 + \alpha_1^2) \\
& (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4)), \frac{1}{288} (36 x_2 x_3^2 \alpha_2^2 \alpha_3 \mu_4 \mu_3 \alpha_1 - 24 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_2 \beta_4 \alpha_1 \\
& - 288 x_3 x_2^2 v_6 \alpha_1^5 + 268 x_3 x_2^2 \alpha_2 \mu_1 \beta_3 \alpha_3^2 \alpha_1 + 208 x_3 x_2^2 \alpha_2 \mu_1 \beta_2 \alpha_1^2 \alpha_3 \\
& + 60 x_3 x_2^2 \alpha_2 \mu_2 \beta_6 \alpha_3^2 \alpha_1 + 192 x_3 x_2^2 \alpha_2 \mu_2 \alpha_3^2 \beta_2 \alpha_1 - 460 x_3 x_2^2 \alpha_2 \beta_6 \alpha_3 \mu_1 \alpha_1^2 \\
& - 140 x_3 x_2^2 \alpha_2 \alpha_3^2 \mu_4 \beta_3 \alpha_1 + 288 x_3 x_2^2 \alpha_2 \beta_1 \mu_2 \alpha_3 \alpha_1^2 - 52 x_3 x_2^2 \alpha_2 \beta_6 \mu_4 \alpha_3 \alpha_1^2 \\
& + 172 x_3 x_2^2 \alpha_3 \mu_4 \beta_3 \alpha_1^3 - 108 x_3 x_2^2 \alpha_2 \mu_2 \beta_5 \alpha_3 \alpha_1^2 - 204 x_3 x_2^2 \alpha_2 \beta_3 \alpha_3 \alpha_1^2 \mu_3 \\
& - 272 x_3 x_2^2 \alpha_2 \beta_2 \mu_4 \alpha_3 \alpha_1^2 - 44 x_3 x_2^2 \alpha_3 \mu_1 \beta_3 \alpha_1^3 - 27 x_2^3 \alpha_2^4 \alpha_3 \gamma_5 \\
& - 312 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_1 \mu_1 \alpha_1 + 60 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_1 \beta_5 \alpha_1 - 60 x_3 x_2^2 \alpha_2^2 \alpha_3 \mu_4 \beta_5 \alpha_1 \\
& - 120 x_3 x_2^2 \alpha_3 \mu_2 \beta_2 \alpha_1^3 + 96 x_2^3 \beta_3 \beta_5 \alpha_1^4 + 144 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_2 \mu_3 \alpha_1
\end{aligned}$$

$$\begin{aligned}
& + 372 x_3 x_2^2 \alpha_3 \mu_2 \beta_6 \alpha_1^3 - 72 x_1 x_3^2 \alpha_2 \alpha_3^2 \mu_2 \mu_3 \alpha_1 - 36 x_1 x_3^2 \alpha_2 \alpha_3 \alpha_1^2 \mu_1 \mu_3 \\
& + 36 x_1 x_3^2 \alpha_2 \alpha_3 \alpha_1^2 \mu_4 \mu_3 + 144 x_1 x_3^2 \rho_3 \alpha_1^5 - 72 x_1 x_3^2 \alpha_2 \alpha_3^2 \mu_4 \mu_1 \alpha_1 \\
& - 36 x_2 x_3^2 \alpha_2^2 \alpha_3 \mu_1 \mu_3 \alpha_1 + 36 x_2 x_3^2 \alpha_2 \alpha_3^2 \mu_1 \mu_2 \alpha_1 + 192 x_1 x_2 x_3 \alpha_2 \alpha_3 v_5 \alpha_1^3 \\
& + 576 x_1 x_2 x_3 \alpha_3 v_6 \alpha_1^4 - 168 x_1 x_2 x_3 \alpha_2 \alpha_3^3 \mu_2 \beta_6 - 36 x_2 x_3^2 \alpha_2 \alpha_3^2 \mu_2 \mu_4 \alpha_1 \\
& + 408 x_1 x_2 x_3 \alpha_2^2 \beta_4 \alpha_1^2 \mu_3 - 9 x_2^3 \alpha_2^3 \alpha_3 \beta_5^2 + 24 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \mu_2 \beta_4 \\
& - 96 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 v_5 \alpha_1 - 44 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \mu_4 \beta_5 + 80 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_1 \mu_1 \\
& + 27 x_2^3 \alpha_2^3 \alpha_3^2 \gamma_2 - 24 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_6 \mu_3 - 208 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_1 \mu_4 \\
& - 12 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \beta_2 \mu_3 + 384 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 v_1 \alpha_1 - 116 x_1 x_2 x_3 \alpha_2^2 \alpha_3^2 \mu_1 \beta_5 \\
& + 96 x_1 x_2 x_3 \alpha_2^2 \alpha_3^3 v_2 - 252 x_1 x_2 x_3 \alpha_3 \mu_4 \beta_2 \alpha_1^3 - 672 x_1 x_2 x_3 \alpha_3 \mu_3 \beta_3 \alpha_1^3 \\
& - 588 x_1 x_2 x_3 \alpha_3 \mu_2 \beta_5 \alpha_1^3 + 336 x_1 x_2 x_3 \alpha_3 \beta_1 \mu_2 \alpha_1^3 - 384 x_1 x_2 x_3 \alpha_3^2 v_3 \alpha_1^3 \\
& - 27 x_2^3 \alpha_2^3 \gamma_5 \alpha_1^2 + 18 x_2^3 \alpha_2 \beta_3^2 \alpha_3^3 - 792 x_1 x_2 x_3 \alpha_3^2 \mu_2 \beta_6 \alpha_1^2 \\
& + 540 x_1 x_2 x_3 \alpha_3^2 \beta_2 \mu_2 \alpha_1^2 - 592 x_1 x_2 x_3 \alpha_3^2 \mu_4 \beta_3 \alpha_1^2 + 96 x_1^2 x_3 v_4 \alpha_1^5 \\
& + 72 x_2^3 \alpha_2 \gamma_7 \alpha_1^4 - 45 x_2^3 \alpha_2^2 \alpha_3^3 \gamma_4 + 624 x_1 x_2 x_3 \alpha_3^3 \mu_2 \beta_3 \alpha_1 \\
& + 464 x_1 x_2 x_3 \alpha_3^2 \beta_3 \mu_1 \alpha_1^2 - 15 x_2^3 \alpha_2^2 \alpha_3^2 \beta_2^2 + 252 x_1 x_2 x_3 \alpha_3 \beta_2 \alpha_1^3 \mu_1 \\
& - 216 x_1 x_2 x_3 \beta_2 \mu_3 \alpha_1^4 + 108 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \mu_3 + 312 x_3 x_2^2 \alpha_2^2 \alpha_3 \beta_1 \mu_4 \alpha_1 \\
& - 24 x_2^3 \alpha_2^2 \beta_5^2 \alpha_1^2 - 54 x_1^3 \alpha_3^2 \gamma_7 \alpha_1^3 - 57 x_1 x_2^2 \alpha_2^2 \alpha_3 \beta_2 \beta_4 \alpha_1 \\
& + 135 x_1 x_2^2 \alpha_2^2 \alpha_3^3 \gamma_8 + 135 x_1 x_2^2 \alpha_2^2 \alpha_3 \beta_1 \beta_5 \alpha_1 - 162 x_1 x_2^2 \alpha_2^2 \gamma_5 \alpha_1^3 \\
& + 102 x_1 x_2^2 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \beta_4 - 188 x_1^2 x_3 \alpha_2 \alpha_3^2 \beta_5 \mu_1 \alpha_1 + 36 x_2^3 \alpha_3 \gamma_4 \alpha_1^4 \\
& + 188 x_1^2 x_3 \alpha_2 \alpha_3^2 \mu_1 \beta_1 \alpha_1 + 432 x_1^2 x_3 \alpha_3^2 \mu_3 \beta_3 \alpha_1^2 - 72 x_1^2 x_3 \alpha_2 \alpha_3^2 \beta_2 \mu_3 \alpha_1 \\
& - 180 x_1^2 x_3 \alpha_2 \alpha_3^2 \beta_4 \mu_2 \alpha_1 + 124 x_1^2 x_3 \alpha_2 \alpha_3 \mu_4 \beta_4 \alpha_1^2 + 30 x_2^3 \alpha_2^4 \beta_4^2 \\
& - 380 x_1^2 x_3 \alpha_2 \alpha_3 \beta_4 \alpha_1^2 \mu_1 + 192 x_1^2 x_3 \alpha_2 \alpha_3^4 v_3 - 252 x_1^2 x_3 \alpha_2 \alpha_3 \mu_3 \beta_1 \alpha_1^2 \\
& - 72 x_2^3 \alpha_3^2 \beta_3^2 \alpha_1^2 - 24 x_1^3 \alpha_3 \beta_1^2 \alpha_1^3 + 252 x_1^2 x_3 \alpha_2 \alpha_3 \mu_3 \beta_5 \alpha_1^2 \\
& + 4 x_1^2 x_3 \alpha_3 \beta_5 \mu_4 \alpha_1^3 + 72 x_1^2 x_3 \alpha_2 \alpha_3^2 \beta_6 \mu_3 \alpha_1 + 316 x_1^2 x_3 \alpha_2 \alpha_3^2 \beta_5 \mu_4 \alpha_1 \\
& - 316 x_1^2 x_3 \alpha_2 \alpha_3^2 \mu_4 \beta_1 \alpha_1 - 324 x_1^2 x_3 \alpha_2^2 \alpha_3 \beta_4 \mu_3 \alpha_1 - 24 x_1^2 x_3 \beta_5 \mu_3 \alpha_1^4 \\
& - 492 x_1 x_2 x_3 \alpha_2 \beta_5 \mu_3 \alpha_1^3 - 64 x_1^2 x_3 \beta_4 \alpha_1^4 \mu_1 + 240 x_1 x_2 x_3 \alpha_2 \mu_3 \beta_1 \alpha_1^3 \\
& + 32 x_1 x_2 x_3 \alpha_2 \mu_4 \beta_3 \alpha_3^3 + 432 x_1 x_2 x_3 \beta_6 \alpha_1^4 \mu_3 - 88 x_1 x_2 x_3 \alpha_2 \mu_4 \beta_4 \alpha_1^3 \\
& + 344 x_1 x_2 x_3 \alpha_2 \beta_4 \alpha_1^3 \mu_1 - 144 x_2^3 \gamma_8 \alpha_1^5 - 24 x_1^3 \beta_5 \beta_4 \alpha_1^4 + 144 x_1^2 x_2 \gamma_6 \alpha_1^5 \\
& - 160 x_1 x_2 x_3 \alpha_2 \beta_3 \alpha_3^3 \mu_1 - 90 x_1^2 x_2 \beta_3^2 \alpha_3^4 - 504 x_1 x_2 x_3 \alpha_3 \beta_6 \alpha_1^3 \mu_1
\end{aligned}$$

$$\begin{aligned}
& -84 x_1 x_2 x_3 \alpha_2 \alpha_3^3 \beta_2 \mu_2 + 504 x_1 x_2 x_3 \alpha_3 \mu_4 \beta_6 \alpha_1^3 - 384 x_1 x_2 x_3 \alpha_2 \alpha_3^3 v_3 \alpha_1 \\
& - 288 x_1 x_2 x_3 \alpha_1^4 \beta_5 \mu_1 + 768 x_1 x_2 x_3 \alpha_2 \alpha_3^2 v_6 \alpha_1^2 - 192 x_1 x_2 x_3 \alpha_2 \alpha_3^2 v_2 \alpha_1^2 \\
& + 384 x_1 x_2 x_3 \alpha_2 \alpha_3 v_1 \alpha_1^3 + 288 x_1 x_2 x_3 v_5 \alpha_1^5 + 45 x_2^3 \alpha_2^3 \alpha_3^2 \gamma_7 + 27 x_1^3 \alpha_2 \alpha_3^4 \gamma_3 \\
& - 27 x_1^3 \alpha_2 \alpha_3^4 \gamma_8 - 12 x_1^3 \alpha_3 \beta_5^2 \alpha_1^3 + 36 x_1^3 \alpha_3 \gamma_1 \alpha_1^4 - 129 x_1^2 x_2 \alpha_2 \alpha_3^2 \beta_2 \alpha_1 \beta_1 \\
& + 84 x_1^2 x_2 \alpha_2 \beta_6 \alpha_3^2 \alpha_1 \beta_1 + 174 x_1^2 x_2 \alpha_2 \beta_4 \beta_2 \alpha_1^2 \alpha_3 - 108 x_1^2 x_2 \alpha_2 \beta_4 \beta_3 \alpha_3^2 \alpha_1 \\
& + 27 x_1^3 \alpha_3^3 \gamma_3 \alpha_1^2 + 84 x_1^2 x_2 \alpha_2 \beta_5 \alpha_3^2 \beta_2 \alpha_1 - 39 x_1^2 x_2 \alpha_2 \beta_5 \beta_6 \alpha_3^2 \alpha_1 \\
& - 84 x_1^2 x_2 \alpha_2 \beta_6 \alpha_1^2 \alpha_3 \beta_4 + 246 x_1^2 x_2 \alpha_2 \beta_5 \alpha_3 \beta_1 \alpha_1^2 - 186 x_1^2 x_2 \alpha_2^2 \alpha_3 \beta_1 \beta_4 \alpha_1 \\
& + 177 x_1^2 x_2 \alpha_2^2 \alpha_3 \beta_5 \beta_4 \alpha_1 - 96 x_1^2 x_2 \alpha_3 \beta_6 \alpha_1^3 \beta_1 + 48 x_1 x_2^2 \alpha_2 \beta_1 \beta_2 \alpha_1^2 \alpha_3 \\
& + 81 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \gamma_1 - 30 x_1^3 \alpha_3^3 \beta_6^2 \alpha_1 + 6 x_1 x_2^2 \alpha_2 \beta_3 \alpha_3^2 \alpha_1 \beta_1 \\
& + 207 x_1 x_2^2 \alpha_2^2 \alpha_3 \gamma_6 \alpha_1^2 - 45 x_1^3 \alpha_3^4 \beta_2 \beta_3 - 45 x_1^3 \alpha_3^3 \beta_2^2 \alpha_1 - 27 x_1^3 \alpha_3^3 \gamma_8 \alpha_1^2 \\
& + 54 x_1^3 \alpha_3^2 \gamma_2 \alpha_1^3 - 24 x_1 x_2^2 \alpha_2 \beta_5 \beta_6 \alpha_1^2 \alpha_3 - 48 x_1 x_2^2 \alpha_2 \beta_6 \alpha_3 \beta_1 \alpha_1^2 \\
& - 105 x_1 x_2^2 \alpha_2 \beta_5 \beta_3 \alpha_3^2 \alpha_1 + 135 x_1 x_2^2 \alpha_2 \beta_2 \beta_6 \alpha_3^2 \alpha_1 + 24 x_1 x_2^2 \alpha_2 \beta_2 \beta_5 \alpha_3 \alpha_1^2 \\
& - 216 x_1 x_2 x_3 \alpha_2^3 \alpha_3 \beta_4 \mu_3 + 45 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_6 - 36 x_1^3 \alpha_3 \gamma_6 \alpha_1^4 + 30 x_1^3 \alpha_3^4 \beta_6 \beta_3 \\
& + 48 x_1^3 \beta_4 \beta_1 \alpha_1^4 - 45 x_1^3 \alpha_2^2 \alpha_3^3 \gamma_1 + 48 x_1^3 \alpha_2 \beta_4^2 \alpha_1^3 - 36 x_2 x_3^2 \alpha_3 \mu_2 \mu_4 \alpha_1^3 \\
& + 72 x_1^3 \gamma_5 \alpha_1^5 - 72 x_1 x_3^2 \mu_3 \alpha_1^4 \mu_1 - 36 x_2 x_3^2 \alpha_2^2 \alpha_1^2 \mu_3^2 + 72 x_2 x_3^2 \alpha_2 \rho_3 \alpha_1^4 \\
& + 72 x_2 x_3^2 \alpha_2^3 \alpha_3 \rho_3 + 36 x_2 x_3^2 \alpha_3^2 \mu_2^2 \alpha_1^2 - 72 x_2 x_3^2 \alpha_2^2 \alpha_3 \rho_2 \\
& + 36 x_2 x_3^2 \alpha_2 \alpha_3^3 \mu_2^2 - 72 x_2 x_3^2 \alpha_3 \rho_2 \alpha_1^4 + 36 x_1 x_3^2 \alpha_2 \alpha_3^2 \mu_4^2 \alpha_1 \\
& - 72 x_1 x_3^2 \alpha_3 \rho_1 \alpha_1^4 + 144 x_1 x_3^2 \alpha_2 \alpha_3^2 \rho_4 \alpha_1^2 - 72 x_1 x_3^2 \alpha_2^2 \alpha_3^3 \rho_1 \\
& + 72 x_1 x_3^2 \alpha_2^2 \alpha_3 \rho_4 + 36 x_1 x_3^2 \alpha_3 \mu_1^2 \alpha_1^3 + 72 x_1 x_3^2 \alpha_3 \rho_4 \alpha_1^4 \\
& - 144 x_1 x_3^2 \alpha_2 \alpha_3^2 \rho_1 \alpha_1^2 + 36 x_1 x_3^2 \alpha_2 \alpha_3^3 \mu_1 \mu_2 + 36 x_1 x_3^2 \alpha_2^2 \alpha_3^2 \mu_1 \mu_3 \\
& + 72 x_1 x_3^2 \mu_4 \mu_3 \alpha_1^4 - 72 x_1 x_3^2 \alpha_2^2 \alpha_3 \mu_3^2 \alpha_1 - 72 x_1 x_3^2 \alpha_3 \alpha_1^3 \mu_2 \mu_3 \\
& - 36 x_1 x_3^2 \alpha_2^2 \alpha_3^2 \mu_4 \mu_3 + 144 x_1 x_3^2 \alpha_2^2 \alpha_3^2 \rho_3 \alpha_1 - 36 x_2 x_3^2 \alpha_2 \mu_3 \alpha_1^3 \mu_1 \\
& + 36 x_2 x_3^2 \alpha_2 \mu_4 \mu_3 \alpha_1^3 - 144 x_2 x_3^2 \alpha_2 \alpha_3^2 \rho_2 \alpha_1^2 - 36 x_2 x_3^2 \alpha_2^3 \alpha_3 \mu_3^2 \\
& + 36 x_2 x_3^2 \alpha_3 \mu_1 \mu_2 \alpha_1^3 + 144 x_2 x_3^2 \alpha_2^2 \alpha_3 \rho_3 \alpha_1^2 - 76 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \beta_6 \mu_4 \\
& - 192 x_3 x_2^2 \alpha_2^2 v_4 \alpha_1^3 - 192 x_3 x_2^2 \alpha_2^2 \alpha_3^2 v_2 \alpha_1 - 148 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \mu_1 \beta_6 \\
& + 96 x_3 x_2^2 \alpha_3 v_3 \alpha_1^4 - 96 x_3 x_2^2 \alpha_2^2 \alpha_3^3 v_3 - 204 x_3 x_2^2 \alpha_3^2 \mu_2 \beta_3 \alpha_1^2 \\
& + 120 x_3 x_2^2 \beta_3 \mu_3 \alpha_1^4 + 216 x_3 x_2^2 \mu_2 \beta_5 \alpha_1^4 + 36 x_1 x_3^2 \alpha_3 \mu_4^2 \alpha_1^3 \\
& + 36 x_1 x_3^2 \alpha_3^2 \mu_1 \mu_2 \alpha_1^2 - 36 x_1 x_3^2 \alpha_3^2 \mu_2 \mu_4 \alpha_1^2 - 72 x_1 x_3^2 \alpha_2 \mu_3^2 \alpha_1^3
\end{aligned}$$

$$\begin{aligned}
& -72 x_1 x_3^2 \alpha_3 \mu_4 \mu_1 \alpha_1^3 - 36 x_1 x_3^2 \alpha_2 \alpha_3^3 \mu_2 \mu_4 + 288 x_1 x_3^2 \alpha_2 \alpha_3 \rho_3 \alpha_1^3 \\
& + 36 x_1 x_3^2 \alpha_2 \alpha_3^2 \mu_1^2 \alpha_1 + 192 x_3 x_2^2 \alpha_2^3 \alpha_3^2 v_1 - 384 x_3 x_2^2 \alpha_2 \alpha_3 v_6 \alpha_1^3 \\
& - 192 x_3 x_2^2 \alpha_2 \alpha_3 v_2 \alpha_1^3 - 104 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \mu_1 \beta_2 - 12 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \beta_3 \mu_3 \\
& + 296 x_3 x_2^2 \alpha_2^2 \mu_1 \beta_4 \alpha_1^2 - 232 x_3 x_2^2 \alpha_2^2 \mu_4 \beta_4 \alpha_1^2 + 384 x_3 x_2^2 \alpha_2^2 \alpha_3 v_5 \alpha_1^2 \\
& + 96 x_3 x_2^2 \alpha_2^3 \alpha_3^2 v_5 + 192 x_3 x_2^2 \alpha_2^2 \alpha_3 v_1 \alpha_1^2 - 84 x_3 x_2^2 \alpha_2^3 \alpha_3 \beta_5 \mu_3 \\
& - 168 x_3 x_2^2 \alpha_2^3 \alpha_3 \beta_1 \mu_3 - 288 x_3 x_2^2 \beta_6 \mu_4 \alpha_1^4 - 16 x_3 x_2^2 \alpha_2^3 \alpha_3 \mu_1 \beta_4 \\
& + 288 x_3 x_2^2 \alpha_2 v_5 \alpha_1^4 - 396 x_3 x_2^2 \alpha_2^2 \beta_5 \mu_3 \alpha_1^2 - 96 x_3 x_2^2 \alpha_2^2 \alpha_3^2 v_6 \alpha_1 \\
& + 40 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \beta_2 \mu_4 - 24 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \beta_1 \mu_2 - 12 x_3 x_2^2 \alpha_2^2 \alpha_3^2 \mu_2 \beta_5 \\
& + 144 x_3 x_2^2 \alpha_2^2 \beta_1 \alpha_1^2 \mu_3 - 252 x_3 x_2^2 \alpha_2 \beta_5 \alpha_1^3 \mu_1 - 192 x_1^2 x_3 \alpha_3^2 v_6 \alpha_1^3 \\
& + 192 x_1^2 x_3 \alpha_3^2 v_2 \alpha_1^3 - 336 x_3 x_2^2 \alpha_2 \mu_2 \beta_4 \alpha_1^3 + 420 x_3 x_2^2 \alpha_2 \beta_6 \mu_3 \alpha_1^3 \\
& - 96 x_1^2 x_3 \alpha_3 v_5 \alpha_1^4 - 168 x_3 x_2^2 \alpha_2 \beta_2 \mu_3 \alpha_1^3 + 32 x_1^2 x_3 \mu_4 \beta_4 \alpha_1^4 + 24 x_1^2 x_3 \mu_3 \beta_1 \alpha_1^4 \\
& + 108 x_3 x_2^2 \alpha_2 \mu_2 \beta_3 \alpha_3^3 + 252 x_3 x_2^2 \alpha_2 \mu_4 \alpha_1^3 \beta_5 + 27 x_2^3 \alpha_2^2 \alpha_3 \gamma_2 \alpha_1^2 \\
& - 280 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \mu_1 \beta_4 \alpha_1 + 120 x_2^3 \alpha_3 \beta_3 \beta_6 \alpha_1^3 - 96 x_1^2 x_3 \alpha_2^2 \alpha_3^3 v_1 \\
& + 132 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \beta_5 \mu_3 \alpha_1 + 117 x_2^3 \alpha_2^2 \alpha_3 \gamma_7 \alpha_1^2 + 536 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \mu_4 \beta_4 \alpha_1 \\
& - 60 x_2^3 \alpha_3 \beta_2 \beta_3 \alpha_1^3 - 384 x_1 x_2 x_3 \alpha_2^2 \alpha_3 \beta_1 \mu_3 \alpha_1 - 240 x_1 x_2 x_3 \mu_2 \beta_4 \alpha_1^4 \\
& + 96 x_1^2 x_3 \alpha_2^2 \alpha_3^3 v_5 + 312 x_3 x_2^2 \alpha_2^3 \beta_4 \alpha_1 \mu_3 + 192 x_1^2 x_3 \alpha_3^3 v_3 \alpha_1^2 \\
& + 80 x_3 x_2^2 \alpha_2^3 \alpha_3 \mu_4 \beta_4 - 312 x_1^2 x_3 \alpha_3^4 \mu_2 \beta_3 + 96 x_1^2 x_3 \alpha_3 v_1 \alpha_1^4 \\
& - 192 x_3 x_2^2 \alpha_2^3 \alpha_3 v_4 \alpha_1 + 9 x_2^3 \alpha_2^3 \alpha_3 \beta_1 \beta_5 + 30 x_2^3 \alpha_2^3 \alpha_3 \beta_6 \beta_4 \\
& + 396 x_1 x_2 x_3 \alpha_2 \mu_3 \beta_2 \alpha_1^2 \alpha_3 + 372 x_1 x_2 x_3 \alpha_2 \mu_4 \alpha_3^2 \beta_2 \alpha_1 + 24 x_2^3 \alpha_2^2 \beta_1 \beta_5 \alpha_1^2 \\
& - 120 x_1 x_2 x_3 \alpha_2 \mu_4 \beta_6 \alpha_3^2 \alpha_1 + 416 x_1 x_2 x_3 \alpha_2 \mu_4 \alpha_3 \beta_1 \alpha_1^2 + 18 x_2^3 \alpha_2^2 \alpha_3^2 \beta_3 \beta_1 \\
& - 288 x_1 x_2 x_3 \alpha_3 v_2 \alpha_1^4 + 192 x_1 x_2 x_3 \alpha_2^3 \alpha_3^2 v_4 + 120 x_1 x_2 x_3 \alpha_2 \beta_6 \alpha_3^2 \alpha_1 \mu_1 \\
& - 90 x_2^3 \alpha_2^2 \alpha_3^2 \gamma_8 \alpha_1 + 408 x_1 x_2 x_3 \alpha_2 \beta_4 \mu_2 \alpha_3 \alpha_1^2 - 192 x_1 x_2 x_3 \alpha_2 v_4 \alpha_1^4 \\
& + 192 x_1 x_2 x_3 \alpha_2^2 \alpha_3^3 v_6 - 54 x_2^3 \alpha_2^2 \alpha_3^2 \gamma_3 \alpha_1 + 15 x_2^3 \alpha_2^2 \alpha_3^2 \beta_2 \beta_6 \\
& - 9 x_2^3 \alpha_2^2 \alpha_3^2 \beta_5 \beta_3 + 63 x_1^2 x_2 \alpha_2^2 \alpha_3^3 \gamma_7 + 9 x_1^2 x_2 \alpha_2^2 \alpha_3^3 \gamma_2 + 21 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_5^2 \\
& + 216 x_1^2 x_2 \alpha_2^2 \beta_4 \alpha_1^2 + 135 x_1^2 x_2 \alpha_2^2 \alpha_3^3 \gamma_5 + 81 x_1^2 x_2 \alpha_2 \gamma_4 \alpha_3^4 \\
& - 54 x_1^2 x_2 \alpha_2^3 \alpha_3 \beta_4^2 + 3 x_1^2 x_2 \alpha_2 \alpha_3^3 \beta_2^2 - 668 x_1 x_2 x_3 \alpha_2 \beta_5 \mu_4 \alpha_3 \alpha_1^2 \\
& - 72 x_1^2 x_2 \alpha_3 \gamma_2 \alpha_1^4 + 216 x_1 x_2^2 \alpha_3 \gamma_8 \alpha_1^4 + 180 x_1 x_2^2 \alpha_2^3 \beta_4^2 \alpha_1 + 63 x_1 x_2^2 \alpha_2^3 \alpha_3^2 \gamma_6 \\
& + 144 x_1 x_2^2 \alpha_2 \gamma_6 \alpha_1^4 + 9 x_1 x_2^2 \alpha_2^2 \alpha_3^3 \gamma_3 - 162 x_1 x_2^2 \alpha_3^2 \gamma_4 \alpha_1^3
\end{aligned}$$

$$\begin{aligned}
& + 180 x_1 x_2^2 \alpha_3^3 \beta_3^2 \alpha_1 - 72 x_1 x_2^2 \alpha_3 \gamma_3 \alpha_1^4 + 30 x_2^3 \alpha_2^3 \beta_1 \beta_4 \alpha_1 - 15 x_2^3 \alpha_2^3 \alpha_3 \beta_2 \beta_4 \\
& - 15 x_2^3 \alpha_2^3 \beta_4 \beta_5 \alpha_1 + 60 x_2^3 \alpha_2 \beta_6^2 \alpha_1^2 \alpha_3 + 30 x_2^3 \alpha_2 \beta_2^2 \alpha_1^2 \alpha_3 - 48 x_2^3 \alpha_2 \beta_2 \beta_5 \alpha_1^3 \\
& + 48 x_2^3 \alpha_2 \beta_5 \beta_6 \alpha_1^3 - 96 x_2^3 \alpha_2 \beta_3 \beta_4 \alpha_1^3 - 234 x_2^3 \alpha_2 \alpha_3 \gamma_8 \alpha_1^3 - 9 x_2^3 \alpha_2 \alpha_3^2 \gamma_4 \alpha_1^2 \\
& - 54 x_2^3 \alpha_2 \alpha_3 \gamma_3 \alpha_1^3 + 240 x_1^2 x_3 \alpha_3 \beta_2 \mu_3 \alpha_1^3 + 81 x_1^2 x_2 \alpha_3^3 \gamma_4 \alpha_1^2 \\
& + 132 x_1^2 x_3 \alpha_3 \mu_2 \beta_4 \alpha_1^3 - 544 x_1 x_2 x_3 \alpha_2 \alpha_3 \mu_1 \alpha_1^2 \beta_1 + 220 x_1 x_2 x_3 \alpha_2 \beta_5 \alpha_3 \mu_1 \alpha_1^2 \\
& - 288 x_1 x_2 x_3 \alpha_2 \alpha_3^2 \beta_1 \mu_2 \alpha_1 + 36 x_1 x_2 x_3 \alpha_2 \alpha_3^2 \mu_2 \beta_5 \alpha_1 - 216 x_1 x_2 x_3 \alpha_2 \beta_6 \alpha_1^2 \alpha_3 \mu_3 \\
& - 372 x_1 x_2 x_3 \alpha_2 \alpha_3^2 \beta_2 \alpha_1 \mu_1 - 48 x_1 x_2 x_3 \alpha_2 \mu_3 \beta_3 \alpha_3^2 \alpha_1 - 9 x_1^3 \alpha_2 \alpha_3^2 \gamma_1 \alpha_1^2 \\
& + 180 x_1^3 \alpha_2^2 \alpha_3^2 \gamma_5 \alpha_1 - 4 x_1^2 x_3 \alpha_2^2 \alpha_3^2 \mu_1 \beta_4 + 18 x_1^3 \alpha_2^2 \alpha_3^2 \beta_1 \beta_4 \\
& + 288 x_1^2 x_3 \alpha_2^2 \alpha_3^2 \nu_4 \alpha_1 - 72 x_1^3 \alpha_2^2 \alpha_3 \beta_4^2 \alpha_1 + 75 x_1^3 \alpha_3^3 \beta_2 \beta_6 \alpha_1 \\
& - 90 x_1^3 \alpha_3^3 \beta_3 \beta_1 \alpha_1 - 120 x_1^3 \alpha_3 \beta_4 \beta_6 \alpha_1^3 + 72 x_1^3 \alpha_3^2 \beta_6 \alpha_1^2 \beta_1 \\
& + 216 x_1^3 \alpha_3^2 \beta_4 \beta_3 \alpha_1^2 - 66 x_1^3 \alpha_3^2 \beta_5 \beta_6 \alpha_1^2 + 36 x_1^3 \alpha_3 \beta_5 \beta_1 \alpha_1^3 \\
& - 102 x_1^3 \alpha_3^2 \beta_2 \alpha_1^2 \beta_1 + 96 x_1^3 \alpha_3^2 \beta_5 \beta_2 \alpha_1^2 + 63 x_1^3 \alpha_2 \alpha_3^2 \beta_5^2 \alpha_1 \\
& + 180 x_1^3 \alpha_3 \beta_2 \beta_4 \alpha_1^3 + 75 x_1^3 \alpha_3^3 \beta_5 \beta_3 \alpha_1 + 48 x_1^2 x_2 \alpha_2 \alpha_3^3 \beta_6^2 \\
& + 48 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_1^2 + 144 x_1^2 x_2 \alpha_3 \gamma_7 \alpha_1^4 + 48 x_1^2 x_2 \alpha_3^2 \beta_6^2 \alpha_1^2 \\
& + 192 x_1^2 x_2 \beta_6 \alpha_1^4 \beta_4 + 48 x_1^2 x_2 \alpha_3^2 \beta_2^2 \alpha_1^2 - 192 x_1^2 x_2 \beta_2 \beta_4 \alpha_1^4 \\
& - 108 x_1^2 x_2 \alpha_2 \gamma_5 \alpha_1^4 + 312 x_1^2 x_3 \alpha_3^3 \mu_4 \beta_3 \alpha_1 + 252 x_1^3 \alpha_2 \alpha_3 \gamma_5 \alpha_1^3 \\
& + 54 x_1^3 \alpha_2 \alpha_3^3 \gamma_2 \alpha_1 - 54 x_1^3 \alpha_2 \alpha_3^3 \gamma_7 \alpha_1 - 27 x_1^3 \alpha_2^2 \alpha_3^2 \beta_5 \beta_4 \\
& + 312 x_1^2 x_3 \alpha_3^3 \mu_2 \beta_6 \alpha_1 - 312 x_1^2 x_3 \alpha_3^3 \beta_3 \mu_1 \alpha_1 - 124 x_1^2 x_3 \alpha_3 \mu_1 \beta_1 \alpha_1^3 \\
& + 36 x_1^2 x_3 \alpha_2^2 \alpha_3^2 \beta_1 \mu_3 + 33 x_1^3 \alpha_2 \alpha_3^3 \beta_1 \beta_2 + 66 x_1^3 \alpha_2 \alpha_3^2 \beta_1^2 \alpha_1 \\
& - 12 x_1^2 x_3 \alpha_2 \beta_4 \mu_3 \alpha_1^3 + 192 x_1^2 x_3 \alpha_2 \alpha_3^3 \nu_2 \alpha_1 + 64 x_1^2 x_3 \alpha_2 \alpha_3^3 \mu_1 \beta_2 \\
& + 39 x_1^3 \alpha_2 \alpha_3^3 \beta_6 \beta_5 - 60 x_1^2 x_3 \alpha_2 \alpha_3^3 \mu_2 \beta_5 - 24 x_1^3 \alpha_2 \alpha_3^3 \beta_2 \beta_5 \\
& - 220 x_1^2 x_3 \alpha_2^2 \alpha_3^2 \mu_4 \beta_4 - 36 x_1^2 x_3 \alpha_2^2 \alpha_3^2 \beta_5 \mu_3 - 240 x_1^2 x_3 \alpha_3 \beta_6 \alpha_1^3 \mu_3 \\
& + 124 x_1^2 x_3 \alpha_3 \beta_5 \mu_1 \alpha_1^3 - 4 x_1^2 x_3 \alpha_3 \mu_4 \beta_1 \alpha_1^3 - 312 x_1^2 x_3 \alpha_3^3 \beta_2 \mu_2 \alpha_1 \\
& + 252 x_1^2 x_3 \alpha_3^2 \mu_2 \beta_5 \alpha_1^2 - 248 x_1^2 x_3 \alpha_3^2 \beta_2 \alpha_1^2 \mu_1 - 48 x_1^3 \alpha_2 \alpha_3^3 \beta_6 \beta_1 \\
& + 9 x_1^3 \alpha_2 \alpha_3^2 \gamma_6 \alpha_1^2 + 60 x_1^2 x_3 \alpha_2 \alpha_3^3 \beta_1 \mu_2 + 128 x_1^2 x_3 \alpha_2 \alpha_3^3 \beta_6 \mu_4 \\
& + 120 x_1^2 x_3 \alpha_2 \alpha_3^3 \beta_3 \mu_3 - 64 x_1^2 x_3 \alpha_2 \alpha_3^3 \mu_1 \beta_6 - 128 x_1^2 x_3 \alpha_2 \alpha_3^3 \beta_2 \mu_4 \\
& + 96 x_1^3 \alpha_2 \alpha_3^3 \beta_3 \beta_4 + 384 x_1^2 x_3 \alpha_2 \alpha_3 \nu_4 \alpha_1^3 - 192 x_1^2 x_3 \alpha_2 \alpha_3^3 \nu_6 \alpha_1 \\
& - 252 x_1^2 x_3 \alpha_3^2 \beta_1 \mu_2 \alpha_1^2 - 184 x_1^2 x_3 \alpha_3^2 \mu_4 \beta_6 \alpha_1^2 + 184 x_1^2 x_3 \alpha_3^2 \mu_4 \beta_2 \alpha_1^2
\end{aligned}$$

$$\begin{aligned}
& -36 x_2^3 \alpha_2^2 \alpha_3 \beta_3 \beta_4 \alpha_1 - 45 x_2^3 \alpha_2^2 \alpha_3 \beta_1 \beta_2 \alpha_1 + 248 x_1^2 x_3 \alpha_3^2 \beta_6 \alpha_1^2 \mu_1 \\
& -90 x_2^3 \alpha_2 \beta_2 \beta_6 \alpha_1^2 \alpha_3 + 108 x_2^3 \alpha_2 \alpha_3 \beta_1 \alpha_1^2 \beta_3 - 18 x_2^3 \alpha_2 \beta_5 \beta_3 \alpha_3 \alpha_1^2 \\
& + 60 x_2^3 \alpha_2^2 \alpha_3 \beta_1 \beta_6 \alpha_1 - 27 x_2^3 \alpha_2^2 \alpha_3 \beta_6 \alpha_1 \beta_5 + 21 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_6 \beta_5 \\
& + 81 x_1 x_2^2 \alpha_2^2 \alpha_3 \gamma_1 \alpha_1^2 - 45 x_1 x_2^2 \alpha_2^2 \alpha_3 \beta_5^2 \alpha_1 - 240 x_1 x_2^2 \alpha_3 \beta_5 \beta_3 \alpha_1^3 \\
& - 276 x_1 x_2^2 \alpha_3^2 \beta_6 \beta_3 \alpha_1^2 + 186 x_1 x_2^2 \alpha_3^2 \beta_2 \beta_3 \alpha_1^2 + 96 x_1 x_2^2 \alpha_3 \beta_3 \alpha_1^3 \beta_1 \\
& + 12 x_2^3 \alpha_2^2 \alpha_3 \beta_2 \beta_5 \alpha_1 + 75 x_2^3 \alpha_2 \alpha_3^2 \beta_2 \beta_3 \alpha_1 - 30 x_2^3 \alpha_2 \alpha_3^2 \beta_3 \beta_6 \alpha_1 \\
& - 45 x_1 x_2^2 \alpha_2 \beta_2^2 \alpha_3^2 \alpha_1 - 39 x_1 x_2^2 \alpha_2 \beta_2 \beta_3 \alpha_3^3 - 192 x_1 x_2^2 \alpha_2 \beta_2 \beta_4 \alpha_1^3 \\
& - 63 x_1 x_2^2 \alpha_2 \alpha_3^2 \gamma_3 \alpha_1^2 + 351 x_1 x_2^2 \alpha_2 \alpha_3^2 \gamma_8 \alpha_1^2 - 162 x_1 x_2^2 \alpha_2 \alpha_3^3 \gamma_4 \alpha_1 \\
& + 228 x_1 x_2^2 \alpha_2^2 \beta_1 \beta_4 \alpha_1^2 + 15 x_1 x_2^2 \alpha_2^3 \alpha_3 \beta_5 \beta_4 + 24 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_2 \beta_5 \\
& + 3 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_1 \beta_2 - 48 x_1 x_2^2 \alpha_2^2 \alpha_3^2 \beta_1 \beta_6 - 210 x_1 x_2^2 \alpha_2^2 \beta_5 \beta_4 \alpha_1^2 \\
& - 162 x_1 x_2^2 \alpha_2^3 \alpha_3 \gamma_5 \alpha_1 + 192 x_1 x_2^2 \alpha_2 \beta_6 \beta_4 \alpha_1^3 - 90 x_1 x_2^2 \alpha_2 \beta_6^2 \alpha_3^2 \alpha_1 \\
& - 6 x_1 x_2^2 \alpha_2 \beta_6 \beta_3 \alpha_3^3 + 27 x_1^2 x_2 \alpha_2^2 \alpha_3 \gamma_5 \alpha_1^2 - 6 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_6 \beta_4 \\
& + 96 x_1^2 x_2 \alpha_3 \beta_5 \beta_6 \alpha_1^3 - 96 x_1^2 x_2 \alpha_3^2 \beta_2 \beta_6 \alpha_1^2 + 90 x_1^2 x_2 \alpha_3^3 \beta_6 \beta_3 \alpha_1 \\
& - 45 x_1^2 x_2 \alpha_3^3 \beta_2 \beta_3 \alpha_1 + 96 x_1^2 x_2 \alpha_3 \beta_2 \alpha_1^3 \beta_1 - 96 x_1^2 x_2 \alpha_3 \beta_5 \beta_2 \alpha_1^3 \\
& + 48 x_1^2 x_2 \alpha_3^2 \beta_3 \beta_1 \alpha_1^2 - 288 x_1^2 x_2 \alpha_3 \beta_4 \beta_3 \alpha_1^3 - 90 x_1^2 x_2 \alpha_2^2 \alpha_3 \beta_1^2 \alpha_1 \\
& - 42 x_1^2 x_2 \alpha_2^3 \alpha_3 \beta_1 \beta_4 - 42 x_1^2 x_2 \alpha_2 \beta_3 \alpha_3^3 \beta_1 + 126 x_1^2 x_2 \alpha_2 \alpha_3 \gamma_6 \alpha_1^3 \\
& + 162 x_1^2 x_2 \alpha_2 \alpha_3 \gamma_1 \alpha_1^3 + 162 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \gamma_1 \alpha_1 - 132 x_1^2 x_2 \alpha_2 \alpha_3 \beta_1^2 \alpha_1^2 \\
& - 114 x_1^2 x_2 \alpha_2 \beta_5^2 \alpha_3 \alpha_1^2 - 69 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_1 \beta_5 - 18 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \gamma_6 \alpha_1 \\
& + 51 x_1^2 x_2 \alpha_2^2 \alpha_3^2 \beta_2 \beta_4 - 144 x_1^3 \alpha_2 \alpha_3 \beta_4 \beta_1 \alpha_1^2 + 30 x_1^3 \alpha_2 \alpha_3^2 \beta_4 \beta_6 \alpha_1 \\
& + 15 x_1^3 \alpha_2 \alpha_3^2 \beta_4 \beta_2 \alpha_1 + 144 x_1^3 \alpha_2 \alpha_3 \beta_4 \beta_5 \alpha_1^2 + 24 x_1^2 x_2 \alpha_3^2 \beta_5 \beta_3 \alpha_1^2 \\
& - 51 x_1^2 x_2 \alpha_2 \alpha_3^3 \beta_2 \beta_6 + 207 x_1^2 x_2 \alpha_2 \alpha_3^2 \gamma_7 \alpha_1^2 - 63 x_1^2 x_2 \alpha_2 \alpha_3^2 \gamma_2 \alpha_1^2 \\
& - 228 x_1^2 x_2 \alpha_2 \beta_5 \beta_4 \alpha_1^3 + 69 x_1^2 x_2 \alpha_2 \beta_5 \beta_3 \alpha_3^3 + 264 x_1^2 x_2 \alpha_2 \beta_4 \beta_1 \alpha_1^3 \\
& - 129 x_1^3 \alpha_2 \alpha_3^2 \beta_5 \beta_1 \alpha_1) / ((\alpha_2 \alpha_3 + \alpha_1^2)(\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4)), 0)
\end{aligned}$$

## Invariants at fixed degree

```

> eval(invariantsdegree);
2
[ Choose a degree invdegree <= invariantsdegree;
> invdegree:=invariantsdegree;

```

$invdegree := 2$

### – Invariant providing map

A map from the space of homogeneous polynomials of  $invdegree$  onto the invariant polynomials of  $invdegree$  is calculated.

```
[ > eval(invpoly);  
       $[\tau(-\tau^2 + \alpha_1^2 + \alpha_2 \alpha_3), (-4 \alpha_2 \alpha_3 + \tau^2 - 4 \alpha_1^2) \tau (\tau^2 - \alpha_1^2 - \alpha_2 \alpha_3)]$   
[ Note that the command InvariantsProvidingPolynomial(invpoly[deg],M); calculates the  
[ invariant providing map for degree deg, if deg<invariantsdegree.  
[ > invpolymap:=InvariantsProvidingPolynomial(invpoly[invdegre  
      e],M);  
       $invpolymap := 8 \alpha_1^2 \alpha_2 \alpha_3 + 4 \alpha_2^2 \alpha_3^2 + 4 \alpha_1^4 + \tau^2 (-5 \alpha_2 \alpha_3 - 5 \alpha_1^2) + \tau^4$ 
```

### – Finding the invariants

A basis of invariants of degree 'invariantsdegree' is calculated using 'invpolymap'. If you already know the dimension of the space of invariants from some theoretical argument, set 'number' to some positive value (in order to save calculation time) and thus only do 'number' tries for invariants. Another call of invariants(...,eval(thistry),...) will then try more invariants...

```
[ > Invarianten:={}: InvZahl:=0: firsttry:=vector(dim,0):  
[ > firsttry[dim]:=invdegree:number:=-1:eval(firsttry);  
       $[0, 0, 2]$   
[ > thistry, Invarianten, InvZahl:=invariants(invpolymap, eval(In  
      varianten), InvZahl, eval(firsttry), number, invdegree, dim, M);  
       $thistry, Invarianten, InvZahl := [3, 0, 0], \{2 \alpha_3 \alpha_1^2 \alpha_2 x_2^2 + 2 \alpha_3^2 \alpha_2^2 x_2^2 - 2 \alpha_2 \alpha_3^3 x_1^2$   
       $- 2 \alpha_1^2 \alpha_3^2 x_1^2 + 4 x_2 \alpha_3 \alpha_1^3 x_1 + 4 x_2 \alpha_3^2 \alpha_2 \alpha_1 x_1, 4 x_3^2 (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4)\},$   
      2
```

If g is an invariant, and c a constant, c\*g is also an invariant. So it is 'legal' to try to simplify further calculations by eliminating parameters.

```
[ > Inva:=vector(InvZahl):  
[ > for i from 1 to InvZahl do  
      Inva[i]:=factor(polynomialsimplify(Invarianten[i],dim));  
      od;
```

$$Inva_1 := -x_1^2 \alpha_3 + x_2^2 \alpha_2 + 2 x_2 \alpha_1 x_1$$
$$Inva_2 := x_3^2$$

## – Reduced vector field

[ We are especially interested in the invariant independent of  $x[3]$ , so set

```
[ > if depends(Inva[1],x[3]) then Inva:=Inva[2] else  
      Inva:=Inva[1] fi;
```

$$Inva := -x_1^2 \alpha_3 + x_2^2 \alpha_2 + 2 x_2 \alpha_1 x_1$$

```
[ > Inva3a:=lieableitung(f[2],Inva):
```

```
[ > Inva3b:=lieableitung(f[3],Inva):
```

```
[ > theta[1]:=simplify(Inva3a/Inva/x[3]);
```

$$\theta_1 := \mu_4 + \mu_1$$

```
[ > pbasis:=vector([eval(x[3]^2*Inva), eval(Inva^2)]):
```

```
[
```

[ and check for linear dependence:

[ > LK:=LinearCoefficients(pbasis, Inva3b, dim);

$$LK := \left[ \rho_1 + \rho_4, -\frac{1}{4}(\alpha_3 \alpha_1 \beta_3 \beta_5 + 2 \alpha_3 \alpha_1 \beta_3 \beta_1 + \alpha_3 \alpha_1 \beta_2 \beta_6 + 2 \alpha_1^2 \beta_2 \beta_1 - 2 \alpha_1^2 \beta_5 \beta_6 \right. \\ \left. + 2 \alpha_2 \alpha_1 \beta_4 \beta_6 - \alpha_2 \alpha_3 \beta_1 \beta_2 - 2 \alpha_2 \alpha_3 \alpha_1 \gamma_2 - 2 \alpha_2 \alpha_3 \alpha_1 \gamma_7 + \alpha_2 \alpha_3 \beta_6 \beta_5 + 2 \alpha_3^2 \beta_6 \beta_3 \right. \\ \left. - \alpha_3 \alpha_1^2 \gamma_3 + \alpha_2 \alpha_1 \beta_2 \beta_4 + \alpha_3^2 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_2^2 - 2 \alpha_3 \alpha_1 \beta_6^2 - 3 \alpha_3 \alpha_1^2 \gamma_8 + \alpha_2 \alpha_1 \beta_1 \beta_5 \right. \\ \left. + \alpha_2 \alpha_1^2 \gamma_6 - 2 \alpha_2 \alpha_1 \beta_1^2 + 3 \alpha_2 \alpha_1^2 \gamma_1 - 2 \alpha_2^2 \beta_1 \beta_4 + \alpha_2 \alpha_1 \beta_5^2 + \alpha_2^2 \alpha_3 \gamma_6 - 3 \alpha_2 \alpha_3^2 \gamma_8 \right. \\ \left. - \alpha_2 \alpha_3^2 \gamma_3 + 3 \alpha_2^2 \alpha_3 \gamma_1 - \alpha_2^2 \beta_5 \beta_4 - 2 \alpha_1^3 \gamma_2 - 2 \alpha_1^3 \gamma_7) / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4) \right]$$

[ > theta[2]:=LK[1];

$$\theta_2 := \rho_1 + \rho_4$$

[ > theta[3]:=LK[2];

$$\theta_3 := -\frac{1}{4}(\alpha_3 \alpha_1 \beta_3 \beta_5 + 2 \alpha_3 \alpha_1 \beta_3 \beta_1 + \alpha_3 \alpha_1 \beta_2 \beta_6 + 2 \alpha_1^2 \beta_2 \beta_1 - 2 \alpha_1^2 \beta_5 \beta_6 + 2 \alpha_2 \alpha_1 \beta_4 \beta_6 \\ - \alpha_2 \alpha_3 \beta_1 \beta_2 - 2 \alpha_2 \alpha_3 \alpha_1 \gamma_2 - 2 \alpha_2 \alpha_3 \alpha_1 \gamma_7 + \alpha_2 \alpha_3 \beta_6 \beta_5 + 2 \alpha_3^2 \beta_6 \beta_3 - \alpha_3 \alpha_1^2 \gamma_3 \\ + \alpha_2 \alpha_1 \beta_2 \beta_4 + \alpha_3^2 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_2^2 - 2 \alpha_3 \alpha_1 \beta_6^2 - 3 \alpha_3 \alpha_1^2 \gamma_8 + \alpha_2 \alpha_1 \beta_1 \beta_5 + \alpha_2 \alpha_1^2 \gamma_6 \\ - 2 \alpha_2 \alpha_1 \beta_1^2 + 3 \alpha_2 \alpha_1^2 \gamma_1 - 2 \alpha_2^2 \beta_1 \beta_4 + \alpha_2 \alpha_1 \beta_5^2 + \alpha_2^2 \alpha_3 \gamma_6 - 3 \alpha_2 \alpha_3^2 \gamma_8 - \alpha_2 \alpha_3^2 \gamma_3 \\ + 3 \alpha_2^2 \alpha_3 \gamma_1 - \alpha_2^2 \beta_5 \beta_4 - 2 \alpha_1^3 \gamma_2 - 2 \alpha_1^3 \gamma_7) / (\alpha_2^2 \alpha_3^2 + 2 \alpha_1^2 \alpha_2 \alpha_3 + \alpha_1^4)$$

[ We obtain a reduced vector field: (the linear invariattn is given by x[3])

Dx1=0

Dx2=(theta[1] x1 + thetap2[ x1^2) x2 + theta[3] x2^2